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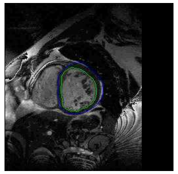
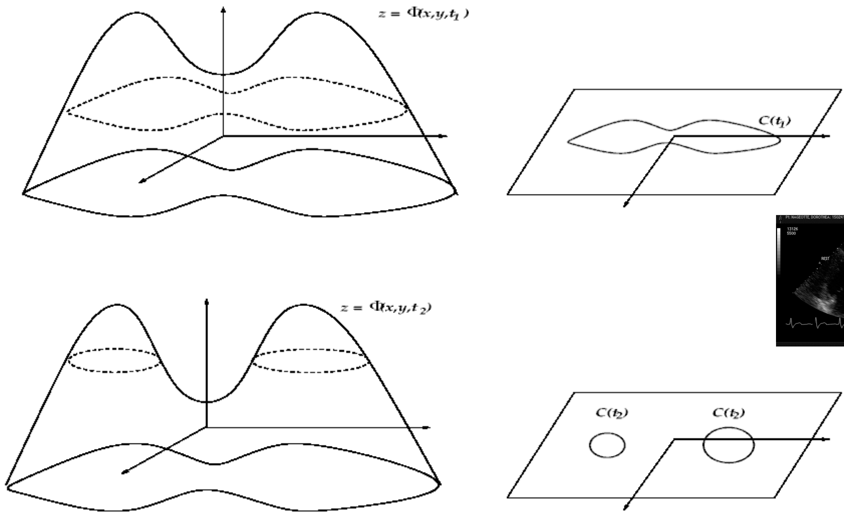
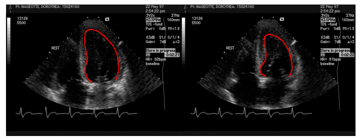
Level Set Methods in Medical Image Analysis: Segmentation







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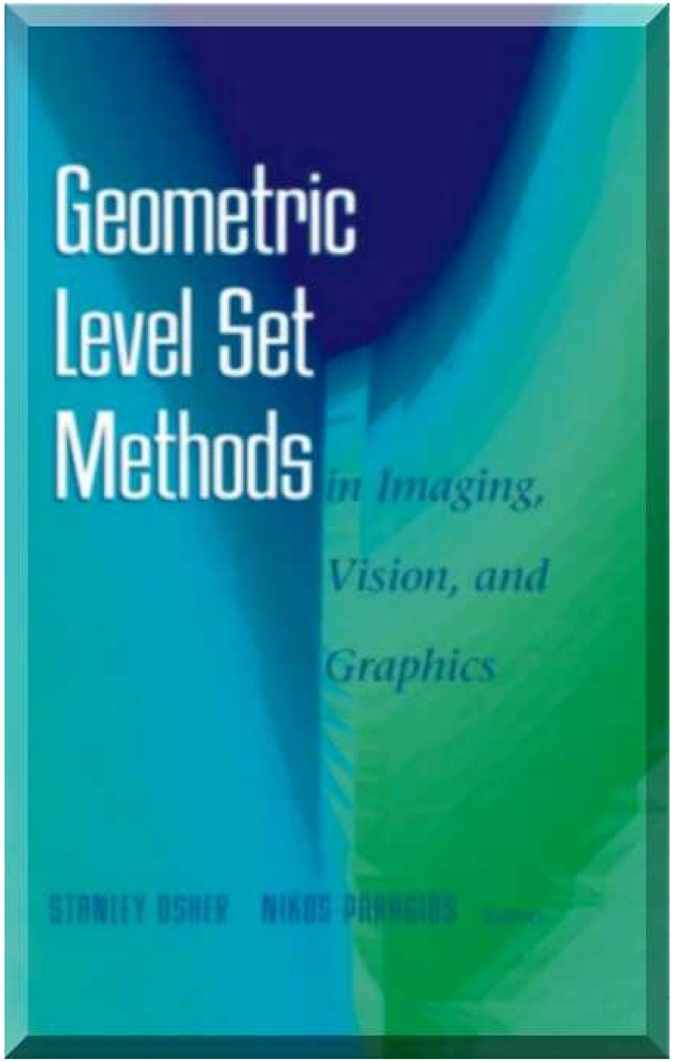

Outline

- Introduction/Motivation
- On the Propagation of Curves
  - The snake model
- The level set method
  - Basic Derivation, algorithms
  - Boundary-driven and Region-driven model free segmentation
- The Level Set Method as a Direct Optimization Space
  - Multiphase Motion
  - Region-driven model free image segmentation
  - Knowledge-based Object Extraction
  - Shape Registration
- Discussion

Motivation

- Image Segmentation and image registration are core components of medical imaging
  - 2002
    - The word "Segmentation" appears 34 times at MICCAI'02 program
    - The word "Registration" appears 22 times at MICCAI'02 program
  - 2003:
    - The word "Segmentation" appears 47 times at MICCAI'03 program ~ 25%
    - The word "Registration" appears 53 times at MICCAI'03 program ~ 25%
  - 2004:
    - The word "Segmentation" appears 51 times at MICCAI'04 program ~ 25%
    - The word "Registration" appears 67 times at MICCAI'04 program ~ 35%

Overview of Segmentation Techniques

- Boundary-driven
  - Edge Detectors (model free)
  - Active Contours/snakes (model free + knowledge-based)
  - Active Shape Models (knowledge-based)
- Region-driven
  - Deformable templates (knowledge-based)
  - Statistical/clustering techniques (model free + knowledge-based)
  - MRF-based techniques (model free)
  - Active Appearance Models (knowledge-based)
- Boundary + Region-driven
  - Active Contours (model free + knowledge-based)
  - Graph-based Techniques (model free)
  - Level Set Methods (model free + knowledge-based)

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On the propagation of Curves

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On the Propagation of Curves

- Snake Model (1987) [Kass-Witkin-Terzopoulos]
  - Planar parameterized curve C:R-->RxR
  - A cost function defined along that curve

$$E[(C)(p)] = \alpha \int_0^1 E_{int}(C(p))dp \; + \; \beta \int_0^1 E_{img}(C(p))dp \; + \; \gamma \int_0^1 E_{con}(C(p))dp$$

  - The internal term stands for regularity/smoothness along the curve and has two components (resisting to stretching and bending)
  - The image term guides the active contour towards the desired image properties (strong gradients)
  - The external term can be used to account for user-defined constraints, or prior knowledge on the structure to be recovered
  - The lowest potential of such a cost function refers to an equilibrium of these terms

Active Contour Components

- The internal term…

$$E_{int}(C(p)) = w_{tension}(C(p)) \left|\frac{\partial C}{\partial p}(p)\right|^2 + w_{stiffness}(C(p)) \left|\frac{\partial^2 C}{\partial p^2}(p)\right|^2$$

  - The first order derivative makes the snake behave as a membrane
  - The second order derivative makes the snake act like a thin plate

- The image term…

$$E_{img}(C(p)) = w_{line}E_{line}(C(p)) + w_{edge}E_{edge}(C(p)) + w_{term}E_{term}(C(p))$$

  - Can guide the snake to
    - Iso-phote $E_{line}(C(p)) = I(C(p))$, edges $E_{edge}(C(p)) = |\nabla I(C(p))|^2$
    - and terminations

- Numerous Provisions…: balloon models, region-snakes, etc…

Optimizing Active Contours

- Taking the Euler-Lagrange equations:

$$\alpha\left(w_{tension}\frac{\partial^2 C}{\partial p^2}(p) - w_{stiffness}\frac{\partial^4 C}{\partial p^4}(p)\right) - \beta\nabla E_{img}(C(p) = 0$$

- That are used to update the position of an initial curve towards the desired image properties
  - Initial the curve, using a certain number of control points as well as a set of basic functions,
  - Update the positions of the control points by solving the above equation
  - Re-parameterize the evolving contour, and continue the process until convergence of the process…

Pros/Cons of such an approach

- Pros
  - Low complexity
  - Easy to introduce prior knowledge
  - Can account for open as well as closed structures
  - A well established technique, numerous publications it works
  - User Interactivity
  - Demetri Terzopoulos is a very good friend
- Cons
  - Selection on the parameter space and the sampling rule affects the final segmentation result
  - Estimation of the internal geometric properties of the curve in particular higher order derivatives
  - Quite sensitive to the initial conditions,
  - Changes of topology (some efforts were done to address the problem)

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Level Set: The basic Derivation

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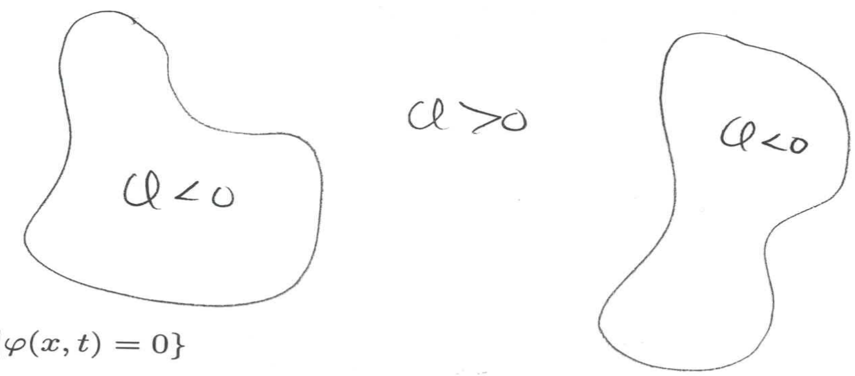
The Level Set Method

- Osher-Sethian (1987)
  - Earlier: Dervieux, Thomassett, (1979, 1980)
- Introduced in the area of fluid dynamics
- Vision and image segmentation
  - Caselles-Catte-coll-Dibos (1992)
  - Malladi-Sethian-Vermuri (1994)
- Level Set Milestones
  - Faugeras-keriven (1998) stereo reconstruction
  - Paragios-Deriche (1998), active regions and grouping
  - Chan-Vese (1999) mumford-shah variant
  - Leventon-Grimson-Faugeras-etal (2000) shape priors
  - Zhao-Fedkiew-Osher (2001) computer graphics

The Level Set Method

- Let us consider in the most general case the following form of curve propagation: $$C(p,t) = F(\mathcal{K})\mathcal{N}$$
- Addressing the problem in a higher dimension...
- The level set method represents the curve in the form of an implicit surface: $$\varphi(x,y,t) : \mathcal{R}^2 \times [0,T) \to \mathcal{R}$$
- That is derived from the initial contour according to the following condition:

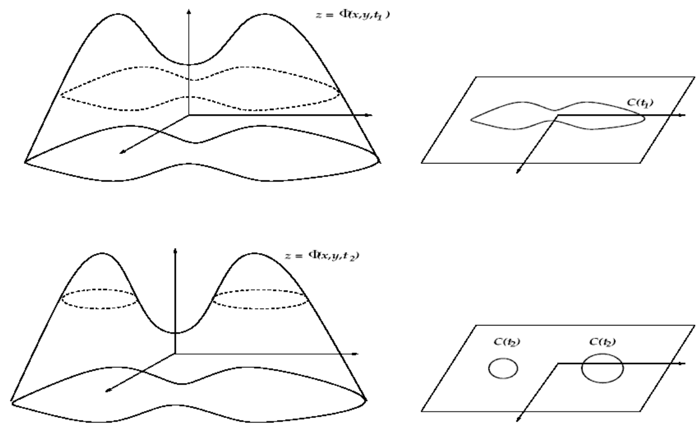
$$C(p,0) = \{(x,y) : \varphi(x,y,0) = 0\}$$



$\{x|\varphi(x,t) = 0\}$ defines $\Gamma(t)$.

The Level Set Method

- Construction of the implicit function



$$C(p,0) = \{(x,y) : \varphi(x,y,0) = 0\}$$
$$C(p,t) = \{(x,y) : \varphi(x,y,t) = 0\}\ ,\ C(t) = \varphi^{-1}(0)$$

- And taking the derivative with respect to time (using the chain rule)

$$\varphi(C(t),t) = 0 \quad \Rightarrow \quad \frac{\partial \varphi}{\partial C} \cdot \underbrace{\frac{\partial C}{\partial t}}_{FN} + \underbrace{\frac{\partial \varphi}{\partial t}}_{\varphi_t} = 0 \qquad (1)$$

- And we are DONE…

The Level Set Method

- Let us consider the arc-length (c) parameterization of the curve, then taking the directional derivative of $\varphi(C(t), t)$, in that direction we will observe no change:

$$\varphi_s = 0 = \varphi_x x_s + \varphi_y y_s = \langle \nabla\varphi, C_s \rangle$$

- leading to the conclusion that the $\nabla\varphi$ is ortho-normal to C where

  the following expression $\left[\mathcal{N} = -\frac{\nabla\varphi}{|\nabla\varphi|}\right]$ for the normal vector

- Embedding the expression of the normal vector to:

$$\varphi(C(t), t) = 0 \;\Rightarrow\; \frac{\partial\varphi}{\partial C} \cdot \underbrace{\frac{\partial C}{\partial t}}_{F\mathcal{N}} + \underbrace{\frac{\partial\varphi}{\partial t}}_{\varphi_t} = 0$$

- the following flow for the implicit function is recovered:

$$\varphi_t = -\langle \nabla\varphi, F(\mathcal{K})\mathcal{N} \rangle = -F(\mathcal{K}) \left\langle \nabla\varphi, -\frac{\nabla\varphi}{|\nabla\varphi|} \right\rangle = F(\mathcal{K})\,|\nabla\varphi| \quad (2)$$

Level Set Method (the basic derivation)

- Where a connection between the curve propagation flow and the flow deforming the implicit function was established
- Given an initial contour, an implicit function is defined and deformed at each pixel according to the equation (2) where the zero-level set corresponds to the actual position of the curve at a given frame
- Euclidean distance transforms are used in most of the cases as embedding function

Overview of the Method

- The level set flow can be re-written in the following form

$$\varphi_t + H(\varphi_x, \varphi_y) = 0,$$

- where H is known to be the Hamiltonian. Numerical approximations is then done according to the form of the Hamiltonian

- Determine the initial implicit function (distance transform)
  - Evolve it locally according to the level set flow
  - Recover the zero-level set iso-surface (curve position)
  - Re-initialize the implicit function and Go to step (1) of the loop

- Computationally expensive

- Open Questions: re-initialization…and numerical approximations

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Implementation Details...

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Level Set Method and Internal Curve Properties

- The normal to the curve/surface can be determined directly from the level set function: $\left[\mathcal{N} = -\frac{\nabla\varphi}{|\nabla\varphi|}\right]$

- The curvature can also be recovered from the implicit function, by taking the second order derivative at the arc length

$$\begin{aligned}\frac{\partial^2\varphi}{\partial s^2} &= \frac{\partial}{\partial s}(\varphi_x x_s + \varphi_y y_s) \\ &= \varphi_{xx}x_s^2 + 2\varphi_{xy}x_s y_s + \varphi_{yy}y_s^2 + \varphi_x x_{ss} + \varphi_y y_{ss} \\ &= \varphi_{xx}x_s^2 + 2\varphi_{xy}x_s y_s + \varphi_{yy}y_s^2 + \langle\nabla\varphi, C_{ss}\rangle \ = \ 0\end{aligned}$$

- Where we observe no variation since the implicit function has constant "zero" values, and given that $[C_{ss} = (x_{ss}, y_{ss}) = \mathcal{K}\mathcal{N}]$ as well as $\left[\mathcal{N} = -\frac{\nabla\varphi}{|\nabla\varphi|}\right]$ one can easily prove that:

$$\mathcal{K} = \frac{\varphi_{xx}\varphi_y^2 - 2\varphi_{xy}\varphi_x\varphi_y + \varphi_{yy}\varphi_x^2}{(\varphi_x^2 + \varphi_y^2)^{3/2}}$$

- That can also be extended to higher dimensions

Examples: Mean/Gaussian Curvature Flow

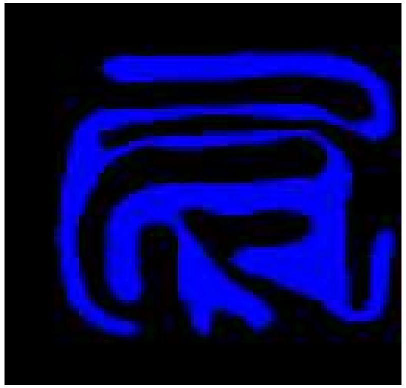
- Minimize the Euclidean length of a curve/surface:

$$C_t = \mathcal{K}\mathcal{N}$$

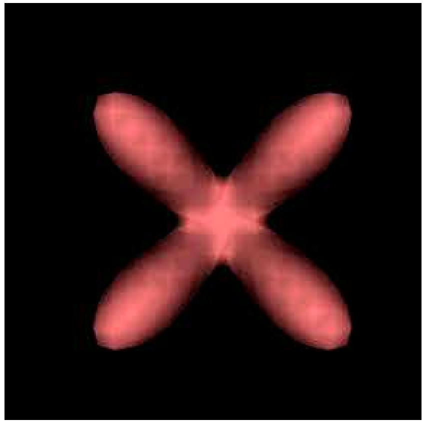
- The corresponding level set variant with a distance transform as an implicit function:

$$\phi_t = \mathcal{K}|\nabla\phi|$$

$$\mathcal{K} = \frac{\varphi_{xx}\varphi_y^2 - 2\varphi_{xy}\varphi_x\varphi_y + \varphi_{yy}\varphi_x^2}{(\varphi_x^2 + \varphi_y^2)^{3/2}}$$



- Things become little bit more complicated at 3D (Gaussian Curvature)

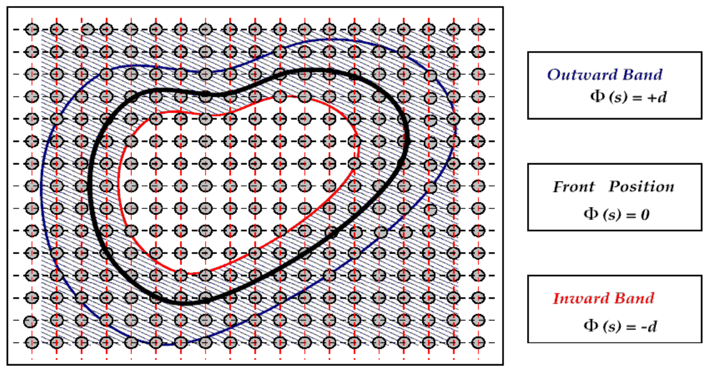


- Results are courtesy Prof. J. Sethian (Berkeley) & G. Hermosillo (INRIA)

From theory to Practice (Narrow Band)

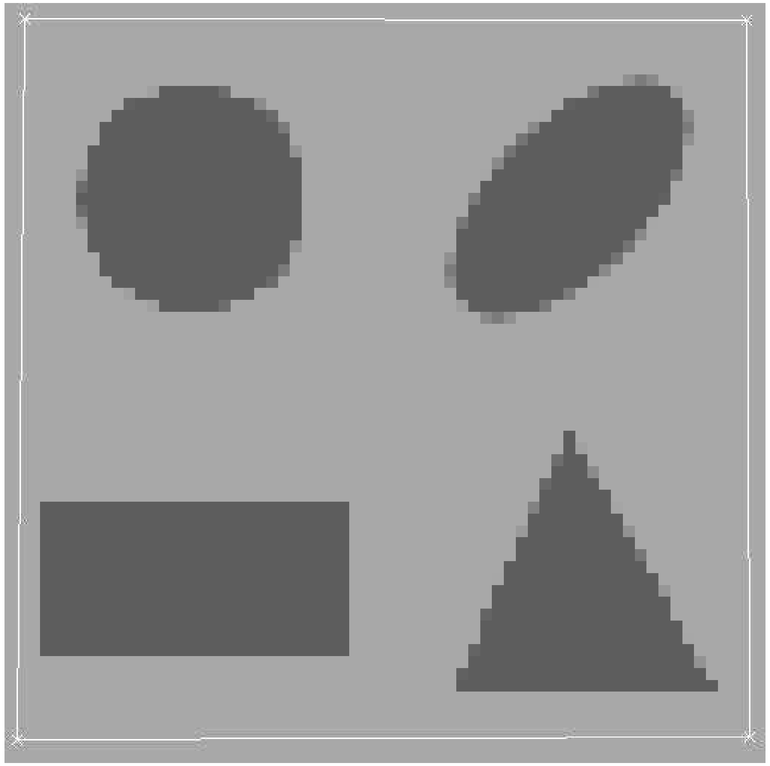
[Chop:93, Adalsteinsson-Sethian:95]

- Central idea: we are interested on the motion of the zero-level set and not for the motion of each iso-phote of the surface
  - Extract the latest position
  - Define a band within a certain distance
  - Update the level set function
  - Check new position with respect the limits of the band
  - Update the position of the band regularly, and re-initialize the implicit function

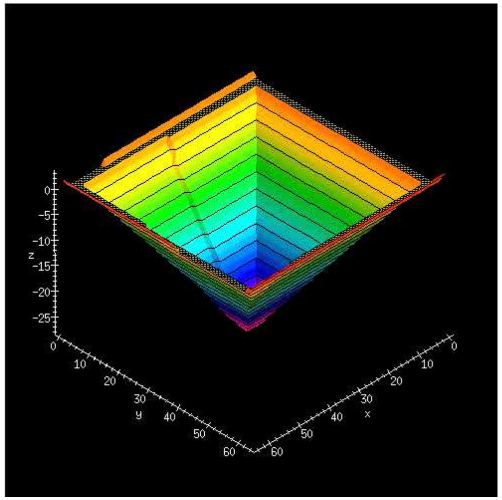


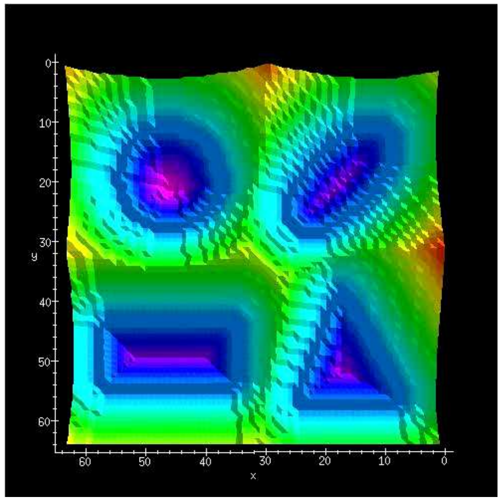
- Significant decrease on the computational complexity, in particular when implemented efficiently and can account for any type of motion flows

Narrow Band (the basic derivation)



Results are courtesy: R. Deriche





Handling the Distance Function

- The distance function has to be frequently re-initialized…
  - Extraction of the curve position & re-initialization:
    - Using the marching cubes one can recover the current position of the curve, set it to zero and then re-initialize the implicit function: the Borgefors approach, the Fast Marching method, explicit estimation of the distance for all image pixels…
  - Preserving the curve position and refinement of the existing function (Susman-smereka-osher:94)

$$\frac{d}{d\tau}\phi_m = \mathrm{sgn}\left(\phi_m^0\right)\left(1 - |\nabla\phi_m|\right)$$

  - Modification on the level set flow such that the distance transform property is preserved (gomes-faugeras:00)
    - Extend the speed of the zero level set to all iso-photes, rather complicated approach with limited added value?

From theory to Practice (Fast Marching)

[Tsitsiklis:93,Sethian:95]

- Central idea: "move" the curve one pixel in a progressive manner according to the speed function while preserving the nature of the implicit function

- Consider the stationary equation $F\ |\nabla T| = 1 \cdot$

- Such an equation can be recovered for all $[\frac{\partial C}{\partial t} = F\mathcal{N}]$ flows where the speed function has one sign (either positive or negative), propagation takes place at one direction

- If T(x,y) is the time when the implicit function reaches (x,y):

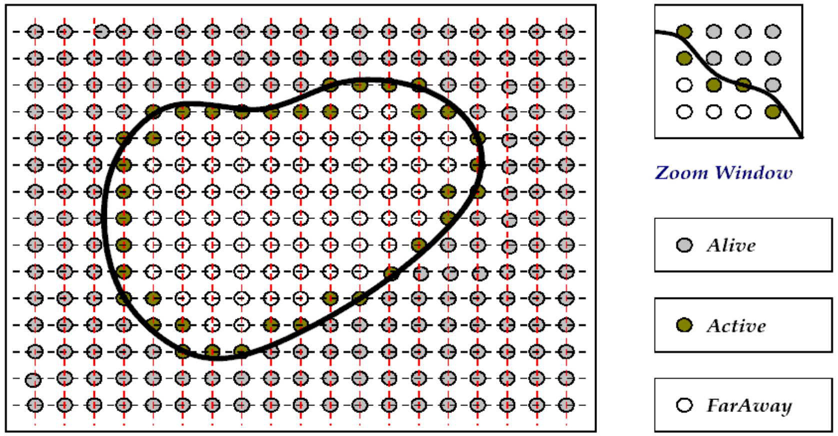
$$\begin{aligned} T(C(p,t)) \triangleq t \quad &\Rightarrow \quad \nabla T \cdot C_t = 1 \\ &\Rightarrow \quad \nabla T \cdot \left(F \frac{\nabla T}{|\nabla T|}\right) = 1 \\ &\Rightarrow \quad F\ |\nabla T| = 1 \end{aligned}$$

Fast Marching (continued)

- Consider the stationary equation $F\ |\nabla T| = 1\cdot$ in its discrete form:

$$\frac{1}{F^2_{\{i,j\}}} = \max\left(D^{-x}_{\{i,j\}}T, 0\right)^2 + \min\left(D^{+x}_{\{i,j\}}T, 0\right)^2 + \max\left(D^{-y}_{\{i,j\}}T, 0\right)^2 + \min\left(D^{+y}_{\{i,j\}}T, 0\right)^2$$

- And using the assumption that the surface propagates in one direction, the solution can be obtained by outwards propagation from the smallest T value…



- active pixels, the curve has already reached them
- alive pixels, the curve could reach them at the next stage
- far away pixels, the curve cannot reach them at this stage

Fast Marching (continued)

INITIAL STEP

- Initialize $[T=0]$ for the all pixels of the front (active), their first order neighbors alive and the rest far away
- For the first order neighbors,
  estimate the arrival time according to: $\left[T_{\{i,j\}} = \frac{1}{F_{\{i,j\}}}\right]$
- While for the rest the crossing time is set to infinity $\left[T_{\{i,j\}} = \infty\right]$

PROPAGATION STEP

- Select the pixel with the lowest arrival time from the alive ones
- Change his label from alive to active and for his first order neighbors:
  - If they are alive, update their T value according to

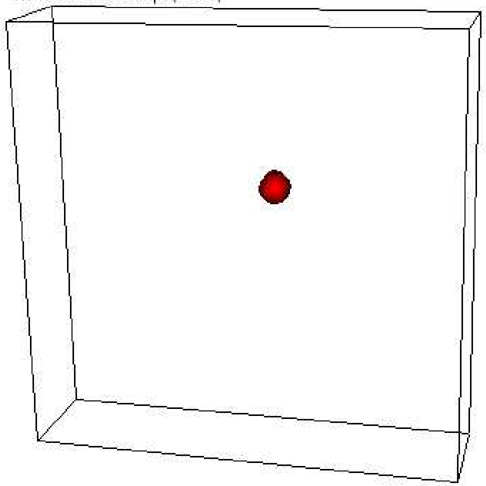
$$\frac{1}{F^2_{\{i,j\}}} = \max\left(D^{-x}_{\{i,j\}}T, 0\right)^2 + \min\left(D^{+x}_{\{i,j\}}T, 0\right)^2 + \max\left(D^{-y}_{\{i,j\}}T, 0\right)^2 + \min\left(D^{+y}_{\{i,j\}}T, 0\right)^2$$

  - If they are far away, estimate the arrival time according to: $\left[T_{\{i,j\}} = \frac{1}{F_{\{i,j\}}}\right]$

Fast Marching Pros/Cons, Some Results

- Fast approach for a level set implementation
- Very efficient technique for re-setting the embedding function to be distance transform
- Single directional flows, great importance on initial placement of the contours
- Absence of curvature related terms or terms that depend on the geometric properties of the curve...
- Results are courtesy: J. Sethian, R. Malladi, T. Deschamps, L. Cohen

Thomas Deschamps(2003)



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Level Sets in imaging and vision…
the edge-driven case

Emigration from Fluid Dynamics to Vision

- (Caselles-Cate-Coll-Dibos:93,Malladi-Sethian-Vemuri:94) have proposed geometric flows to boundary extraction

$$\phi_t(;) = g(;)\,(F_A(;) + F_I(;))\,|\nabla \phi|$$

- Where g(;) is a function that accounts for strong image gradients

$$g(;) = \frac{1}{1 + |\nabla G_s * I(;)|}$$

Malladi-Sethian-Vemuri:94

- And the other terms are application specific…that either expand or shrink constantly the initial curve

- Distance transforms have been used as embedding functions

Geodesic Active Contours

[Caselles-Kimmel-Sapiro:95, Kichenassamy-Kumar-etal95]

- Connection between level set methods and snake driven optimization

- The geodesic active contour consists of a simplified snake model without second order smoothness

$$E[(C)(p)] = \alpha \int_0^1 \left| \frac{\partial C}{\partial p}(p) \right|^2 dp \; - \; \beta \int_0^1 |\nabla I(C(p))| \, dp$$

- That can be written in a more general form as

$$E[(C)(p)] = \alpha \int_0^1 \left| \frac{\partial C}{\partial p}(p) \right|^2 dp \; + \; \beta \int_0^1 g\left(|\nabla I(C(p))|\right)^2 dp$$

- Where the image metric has been replaced with a monotonically decreasing function: $g(;) = \dfrac{1}{1 + |\nabla G_s * I(;)|}$

Geodesic Active Contours

[Caselles-Kimmel-Sapiro:95, Kichenassamy-Kumar-etal95]

- Leading to the following more general framework…

$$E[(C)(p)] = \alpha \int_0^1 \left|\frac{\partial C}{\partial p}(p)\right|^2 dp + (1-\alpha)\int_0^1 g\left(|\nabla I(C(p))|\right)^2 dp$$
$$= \int_0^1 \left(E_{int}(C(p)) + E_{ext}(C(p))\right) dp$$

- One can assume that smoothness as well as image terms are equally important and with some “basic math”

$$E[(C)(p)] = \underbrace{\int_0^L g\left(|\nabla I(C(s))|\right) ds}_{Geodesic\ Active\ Contour} = \int_0^1 \underbrace{g\left(|\nabla I(C(p))|\right)}_{attraction\ term} \underbrace{\left|\frac{\partial C}{\partial p}(p)\right|}_{regularity\ term} dp$$

- That seeks a minimal length geodesic curve attracted by the desired image properties…

Geodesic Active Contours

- That when minimized leads to the following geometric flow:

$$\frac{\partial C}{\partial t} = \underbrace{g(|\nabla I|)\mathcal{K}\mathcal{N}}_{boundary\ force} - \underbrace{(\nabla g(|\nabla I|) \cdot \mathcal{N})\mathcal{N}}_{refinement\ force}$$

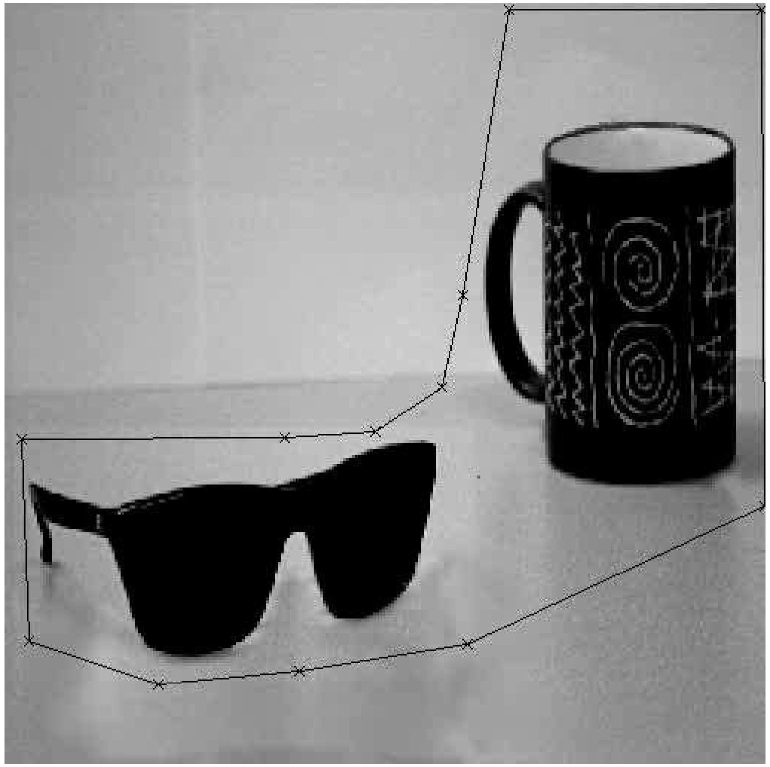
  - Data-driven constrained by the curvature force
  - Gradient driven term that adjusts the position of the contour when close to the real Object boundaries...

- By embedding this flow to a level set framework and using a distance transform as implicit function,

$$\phi_t(p)) = g(p)\mathcal{K}(p)|\nabla\phi(p)| + \nabla g(p) \cdot \nabla\phi(p)$$

Geodesic Active Contours…

- That has an extra term when compared with the flow proposed by Malladi-Sethian-Vemuri.
- Single directional flow…requires the initial contour to either enclose the object or to be completely inside…

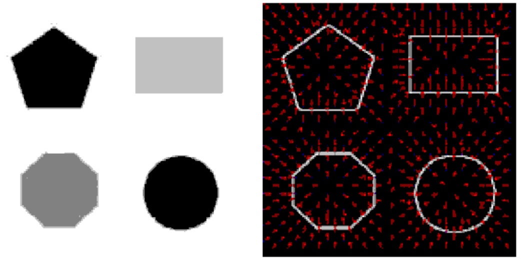


Results are courtesy: R. Deriche

Gradient Vector Flow Geometric Contours

[paragios-mellina-ramesh:01]

- Initial conditions are an issue at the active contours since they are propagated mainly at one direction



- Region terms (later introduced) is a mean to overcome this limitation…

- an alternative is somehow to extend the boundary-driven speed function to account for directionality, thus recovering a field (u,v)

- One can estimate this field close to the object boundaries…where

  - The image gradient at the boundaries is tangent to the curve
  - While the inward normal normal points towards the object boundaries

Gradient Vector Flow Geometric Contours

[paragios-mellina-ramesh:01]

- Let (f) be a continuous edge detector with values close to 1 at the presence of noise and 0 elsewhere...

- The flow can be determined in areas with important boundary information (Important f)

- And areas where there changes on f, |Gradient(f)|

$$E(u,v) = \int\int \alpha \underbrace{(u_x^2 + u_y^2 + v_x^2 + v_y^2)}_{diffusion} + \underbrace{f|\nabla f|^2|(u,v) - \nabla f|}_{field\ formation\ term} \, d\Omega$$

- While elsewhere recovering such a field is not possible and the only way to be done is through diffusion

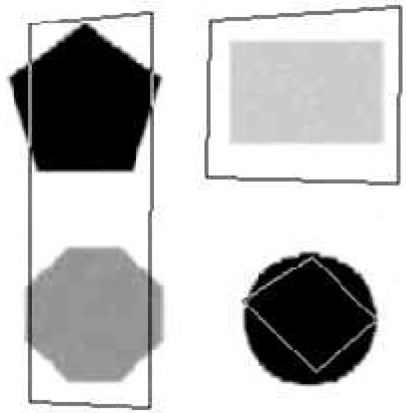
- This can be done through an approximation of image gradient at the edges and diffusion of this information for the rest of the image plane

Gradient Vector Flow Geometric Contours

- This flow can be used within a geometric flow towards image segmentation…
  - The direction of the propagation should be the same with the one proposed by the recovered flow, therefore one can penalize the orientation between these two vectors.

$$\phi_\tau(;) = \beta g(;)\mathcal{K}(;)|\nabla\phi(;)| + (1-\beta)\left(\frac{u(;)}{\sqrt{u^2+v^2}}, \frac{v(;)}{\sqrt{u^2+v^2}}\right)\cdot\nabla\phi(;)$$

  - That is integrated within the classical Geodesic active contour equation and is implemented using the level set function using the Additive Operator Splitting



  - The inner product between the curve normal and the vector field guides the curve propagation

Additive Operator Splitting

[Weickert:98, Goldenberg-Kimmel:01]

- Introduced for fast non-linear diffusion
- Applied to the flow of the geodesic active contour

$$\phi_t = \operatorname{div}\left(g(|\nabla I|) \frac{\nabla \phi}{|\nabla \phi|} \right) |\nabla \phi|$$

- Where one can consider a signed Euclidean distance function to be the implicit function, leading to: $\phi_t = \operatorname{div}\left(g(|\nabla I|) \nabla \phi \right)$
- That can be written as:

$$\operatorname{div}\left(g(|\nabla I|) \nabla \phi \right) = \underbrace{\partial_x (g(|\nabla I|) \nabla \phi_x)}_{A_x(\phi^k)} + \underbrace{\partial_y (g(|\nabla I|) \nabla \phi_y)}_{A_y(\phi^k)}$$

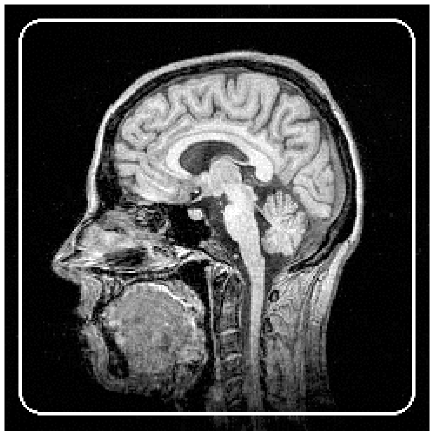
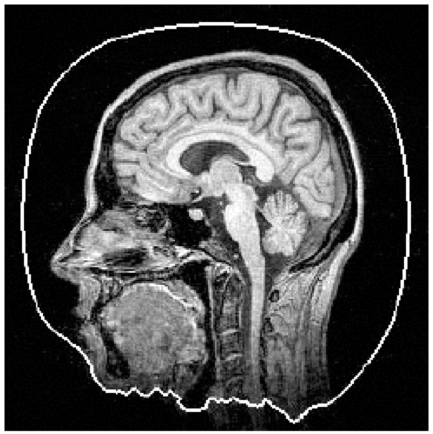
- That can be solved in an explicit form:
- Or a semi-implicit one: $\phi^{k+1} = \left[I + \tau \left[A_x(\phi^k) + A_y(\phi^k) \right] \right] \phi^k$

Additive Operator Splitting (Weickert:02)

- Or in a semi-implicit one

$$\phi^{k+1} = \frac{1}{2}\left[I - 2\tau A_x(\phi^k)\right]^{-1}\phi^k + \frac{1}{2}\left[I - 2\tau A_y(\phi^k)\right]^{-1}\phi^k$$

- That refers to a triagonal system of equations and can be done using the Thomas algorithm…at O(N) and has to be done once…


Some Comparison (Weickert:02)

geometric model (explicit scheme)

image	τ	k	iterations	CPU time	distance
synthetic	0.25	-0.1	20200	49.0 s	0
hall-and-monitor	0.25	-0.1	20000	324.5 s	0
medical	0.25	-0.1	6600	126.3 s	0.01

geometric model (AOS scheme)

image	τ	k	iterations	CPU time	distance
synthetic	5.0	-0.1	950	7.4 s	0.75
hall-and-monitor	5.0	-0.1	1040	54.0 s	0.87
medical	5.0	-0.1	370	25.0 s	0.48

geodesic model (explicit scheme)

image	τ	k	iterations	CPU time	distance
synthetic	0.25	-0.02	10400	36.9 s	0
hall-and-monitor	0.25	-0.02	30800	634.9 s	0
medical	0.25	-0.05	12200	306.1 s	0.01

geodesic model (AOS scheme)

image	τ	k	iterations	CPU time	distance
synthetic	5.0	-0.02	480	4.2 s	1
hall-and-monitor	5.0	-0.02	1390	70.2 s	1.79
medical	5.0	-0.05	640	36.8 s	1.32

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Level Sets in imaging and vision…
the region-driven case

The Mumford-Shah framework

[chan-vese:99, yezzi-tsai-willsky-99]

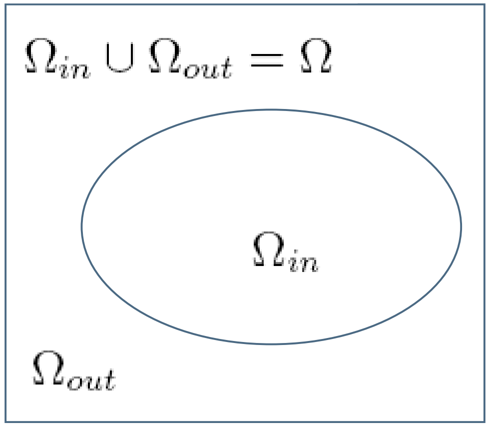
- The original Mumford-Shah framework aims at partitioning the image into (multiple) classes according to a minimal length curve and reconstructing the noisy signal in each class

$$E(C,u) = \alpha \int\int_{\Omega} (I-u)^2 d\omega + \beta \int |C'| dc + \gamma \int\int_{\Omega - C} |\nabla u| d\omega$$

- Let us consider - a simplified version - the binary case and the fact that the reconstructed signal is piece-wise constant

$$E(C,\mu_{in},\mu_{out}) = \alpha \int\int_{\Omega_{in}} (I-\mu_{in})^2 d\omega + \alpha \int\int_{\Omega_{out}} (I-\mu_{out})^2 d\omega + \beta \int |C'| dc$$

- Where the objective is to reconstruct the image, using the mean values for the inner and the outer region



- Tractable problem, numerous solutions…

The Mumford-Shah framework

[chan-vese:99, yezzi-tsai-willsky-99]

- Taking the derivatives with respect to piece-wise constants, it straightforward to show that their optimal value corresponds to the means within each region:

$$\mu_{in} = \frac{\int\int_{\Omega_{in}} I(\omega)d\omega}{\int\int_{\Omega_{in}} d\omega}$$

- While taking the derivatives with respect and using the stokes theorem, the following flow is recovered for the evolution of the curve:

$$C_t = \alpha\left((I-\mu_{in})^2 - (I-\mu_{out})^2\right)\mathcal{N} + \beta\mathcal{K}\mathcal{N}$$

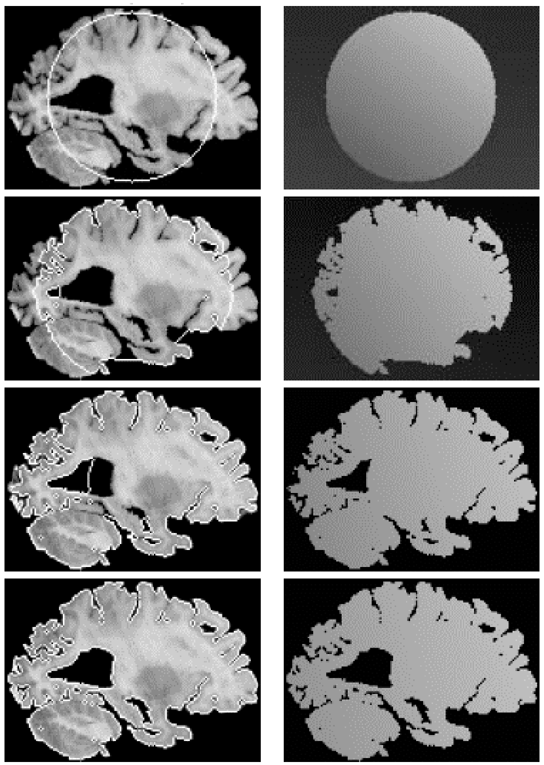
  - *An adaptive (directional/magnitude)-wise balloon force*
  - *A smoothness force aims at minimizing the length of the partition*

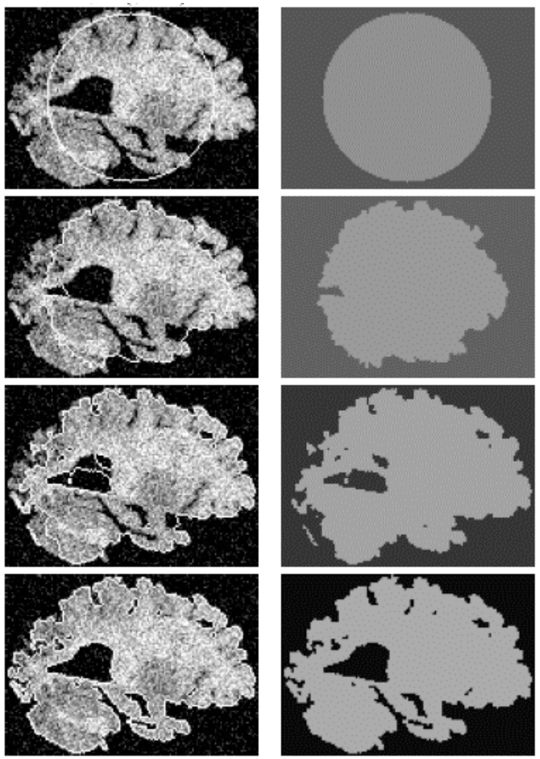
- That can be implemented in a straightforward manner within the level set approach

$$\phi_t = \alpha\left((I-\mu_{in})^2 - (I-\mu_{out})^2\right)|\nabla\phi| + \beta\mathcal{K}|\nabla\phi|$$

The Mumford-Shah framework – Criticism & Results

- Account for multiple classes?
- Quite simplistic model, quite often the means are not a good indicator for the region statistics
- Absence of use on the edges, boundary information



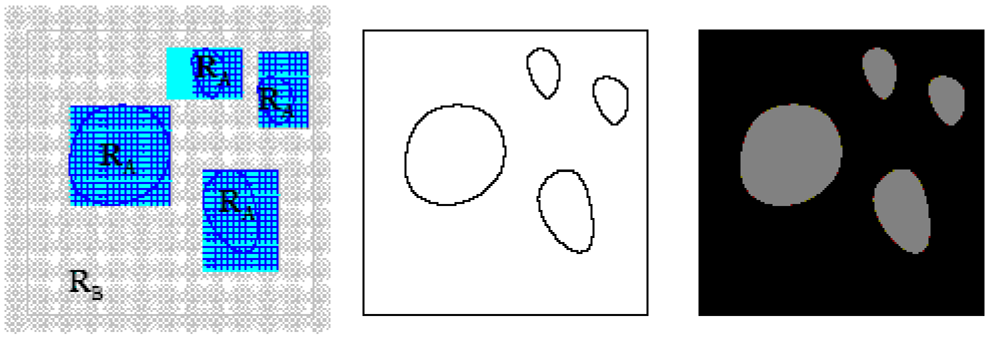


Geodesic Active Regions

[paragios-deriche:98]

- Introduce a frame partition paradigm within the level set space that can account for boundary and global region-driven information

- KEY ASSUMPTIONS
  - Optimize the position and the geometric form of the curve by measuring information along that curve, and within the regions that compose the image partition defined by the curve:



(input image) (boundary) (region)

Geodesic Active Regions

- We assume that prior knowledge on the positions of the objects to be recovered is available - $[p_C()]$ - as well as on the expected intensity properties of the object and the background $[\,p_A(), p_B()\,]$

$$E(\partial\mathcal{R}) = \underbrace{\alpha \int_0^1 \overbrace{g\left(\underbrace{p_C(I(\partial\mathcal{R}(c)))}_{boundary\ probability}\right)}^{boundary\ attraction} \overbrace{\left|\partial\dot{\mathcal{R}}(c)\right|}^{regularity} dc}_{Boundary\ Term}$$

$$\underbrace{-(1-\alpha)\overbrace{\iint_{\mathcal{R}_A} \mathbf{log}\left[\underbrace{p_A(I(x,y))}_{h_A\ probability}\right] dxdy}^{\mathcal{R}_A\ fitting\ measurement} -(1-\alpha)\overbrace{\iint_{\mathcal{R}_B} \mathbf{log}\left[\underbrace{p_B(I(x,y))}_{h_B\ probability}\right] dxdy}^{\mathcal{R}_B\ fitting\ measurement}}_{Region\ Term}$$

Geodesic Active Regions

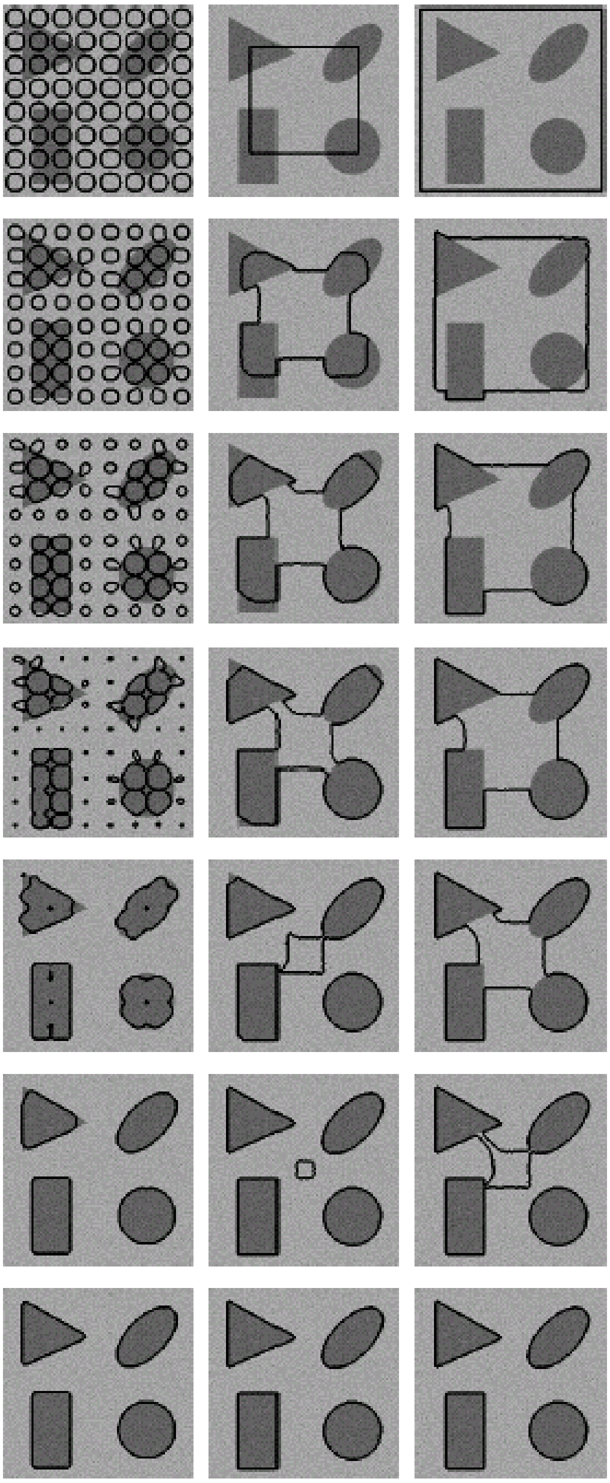
- Such a cost function consists of:
  - The geodesic active contour
  - A region-driven partition module that aims at separating the intensities properties of the two classes (see later analogy with the Mumford-Shah)

- And can be minimized using a gradient descent method leading to:

$$\frac{\partial}{\partial t}u = \left[\underbrace{\alpha \mathbf{log} \left(\frac{\overbrace{p_B(I(u))}^{h_B\ probability}}{\underbrace{p_A(I(u))}_{h_A\ probability}} \right)}_{region-based\ force} + \underbrace{(1-\alpha)\left(g(p_C(I(u)))\mathcal{K}(u) - \nabla g(p_C(I(u))) \cdot \mathcal{N}(u)\right)}_{boundary-based\ force} \right] \mathcal{N}(u)$$

- Which can be implemented using the level set method as follows…

$$\phi_t = \alpha \log\left(\frac{p_B(I)}{p_A(I)}\right)|\nabla\phi| + (1-\alpha)\left(g\mathcal{K}|\nabla\phi| + \nabla\phi\nabla g\right)$$



Geodesic Active Regions

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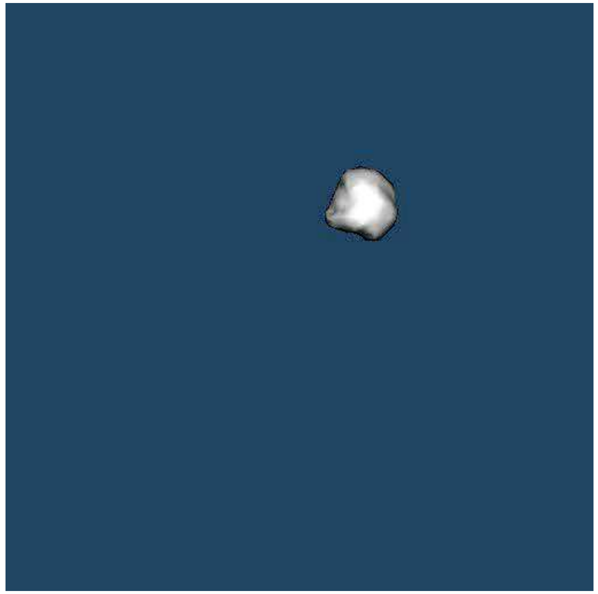
Some Results…



...REMINDER...

Level Set & Geometric Flows

- While evolving moving interfaces with the level set method is quite attracting, still it has the limitation of being a static approach
  - The motion equations are derived somehow,
  - The level set is used only as an implementation tool…

$$C(p,t) = F(\mathcal{K})\mathcal{N}$$

$$\varphi_t = -\langle \nabla\varphi, F(\mathcal{K})\mathcal{N}\rangle = -F(\mathcal{K})\left\langle \nabla\varphi, -\frac{\nabla\varphi}{|\nabla\varphi|}\right\rangle = F(\mathcal{K})\,|\nabla\varphi|$$

  - That is equivalent with saying that the problem has been somehow already solved…since there is not direct connection between the approach and the level set methodology

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Level Set: Optimization space

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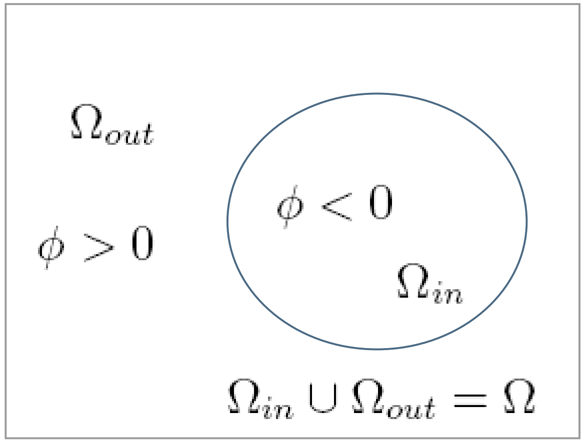
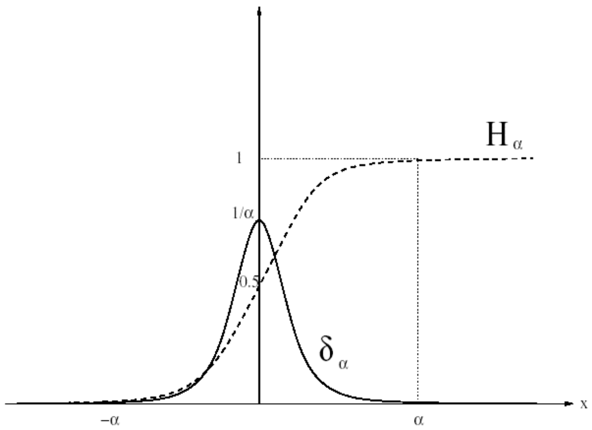
Level Set Dictionary

- Let us consider distance transforms as embedding function

$$\phi(s) = \begin{cases} -D(s,C), & s \in \Omega_{in} \\ 0), & s \in C \\ -D(s,C), & s \in \Omega_{out} \end{cases}$$



- Then following ideas introduced in [evans-gariepy:96], one can introduce the Dirac distribution



$$\delta_\alpha(\phi) = \begin{cases} 0 & , |\phi| > \alpha \\ \frac{1}{2\alpha}\left(1+\cos\left(\frac{\pi\phi}{\alpha}\right)\right) & , |\phi| < \alpha \end{cases}$$

$$H_\alpha(\phi) = \begin{cases} 1 & , \phi > \alpha \\ 0 & , \phi < -\alpha \\ \frac{1}{2}\left(1+\frac{\phi}{\alpha}+\frac{1}{\pi}\sin\left(\frac{\pi\phi}{\alpha}\right)\right) & , |\phi| < \alpha \end{cases}$$

Level Set Dictionary

- Using the Dirac function and integrating within the image domain, one can estimate the length of the curve:

$$|C| = \int\int \delta_\alpha(\phi)|\nabla\phi| d\Omega$$

- While integrating the Heaviside Distribution within the image domain

$$|\Omega_{in}| = \int\int H_\alpha(\phi) d\Omega$$

- Such observations can be used to define regional partition modules as follows according to some descriptors

$$E_{regional}(\phi) = \underbrace{\int\int_\Omega H_\alpha(\phi) r_O(;)\ d\Omega}_{object} + \underbrace{\int\int_\Omega (1 - H_\alpha(\phi)) r_B(;)\ d\Omega}_{background}$$

- That can be optimized with respect to the level set function (implicitly with respect to a curve position)

Level Set Optimization

$$\frac{\partial}{\partial\phi}E_{regional} = \frac{\partial}{\partial\phi}\int\int H_\alpha(-\phi) r_O d\Omega + \frac{\partial}{\partial\phi}\int\int (1-H_\alpha(-\phi)) r_B d\Omega$$
$$= \frac{\partial H_\alpha(-\phi)}{\partial\phi} r_O + H_\alpha(-\phi)\frac{\partial r_O}{\partial\phi} + \frac{\partial(1-H_\alpha(-\phi))}{\partial\phi} r_B + (1-H_\alpha(-\phi))\frac{\partial r_B}{\partial\phi}$$

- And given that : $\frac{\partial H_\alpha(-\phi)}{\partial\phi} = -\delta(-\phi), \quad \frac{\partial r_B}{\partial\phi} = \frac{\partial r_O}{\partial\phi} = 0$
- An adaptive (directional & magnitude wise) flow is recovered for the propagation of an initial surface towards a partition that is optimal according to the regional descriptors…

$$\frac{\partial}{\partial\tau}\phi = \delta_\alpha(\phi)(r_B(;) - r_O(;))$$

- The same idea can be used to introduce contour-driven terms…

$$E_{geodesic}(\phi) = \int\int_\Omega \delta_\alpha(\phi) b(;) |\nabla\phi| \, d\Omega$$

Level Set Optimization

and optimize them directly on the level set space

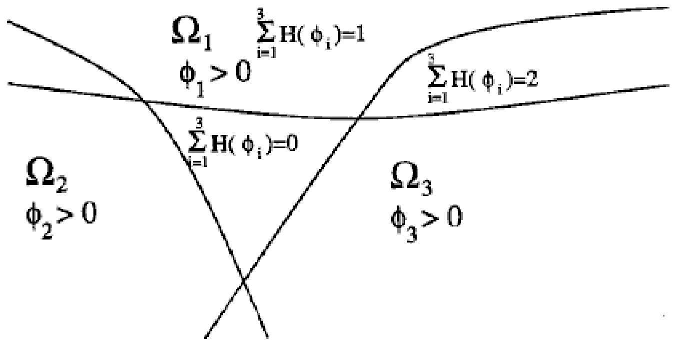
- Curve-driven terms: $\frac{\partial}{\partial \tau}\phi = \delta_\alpha(\phi)\mathrm{div}\left(b(;)\frac{\nabla\phi}{|\nabla\phi|}\right)$

- Global region-driven terms: $\frac{\partial}{\partial \tau}\phi = \delta_\alpha(\phi)(r_B(;) - r_O(;))$

- According to some image metrics…defined along the curve and within the regions obtained through the image partition according to the position of the curve, that can be multi-component but is representing only one class

Multiphase Motion [zhao-chan-merinman-osher:96]

- Up to now statistics and image information have been used to partition image into two classes,
- Often, we need more than object/background separation, and therefore the case of multi-phase motion is to be considered...
- N objects/curves, represented by N level set functions
  - How to deal with occlusions, one image pixel cannot be assigned to more than one curve...
  - How to constrain the solution such that the obtained partition consists of all image data



Multi-Phase Motion (continued)

- For each class, boundary, smoothness as well as region components can be considered

$$E_R(\phi_1, \phi_2, ..., \phi_N) = \sum_{i=1}^{N} u_i \int\int H_\alpha(\phi_i(;))r_i(;)d\Omega$$

$$E_B(\phi_1, \phi_2, ..., \phi_N) = \sum_{i=1}^{N} w_i \int\int \delta_\alpha(\phi_i(;))b_i|\nabla\phi_i(;)|d\Omega$$

- Subject to the constraint at each pixel: $\sum_{i=1}^{N} H_\alpha(\phi_i(;))) = 1$

- a hard and local constraint difficult to be imposed that could be replaced with a more convenient

$$\int\int \left(\sum_{i=1}^{N} H_\alpha(\phi_i(;))) - 1\right)^2 d\Omega = \epsilon$$

- That can be optimized through Lagrange multipliers method...

Multiphase Motion & Mumford-Shah

[samson-aubert-blanc-feraud:99]

- Image Segmentation and Signal Reconstruction (direct application of the (zhao-chan-merinman-osher:96) within the Mumford Shah formulation…)

  - Separate the image into regions with consistent intensity properties
  - Recover a Gaussian distribution that expresses the intensity properties of each class, or force the intensity properties of each class to follow some predefined image characteristics

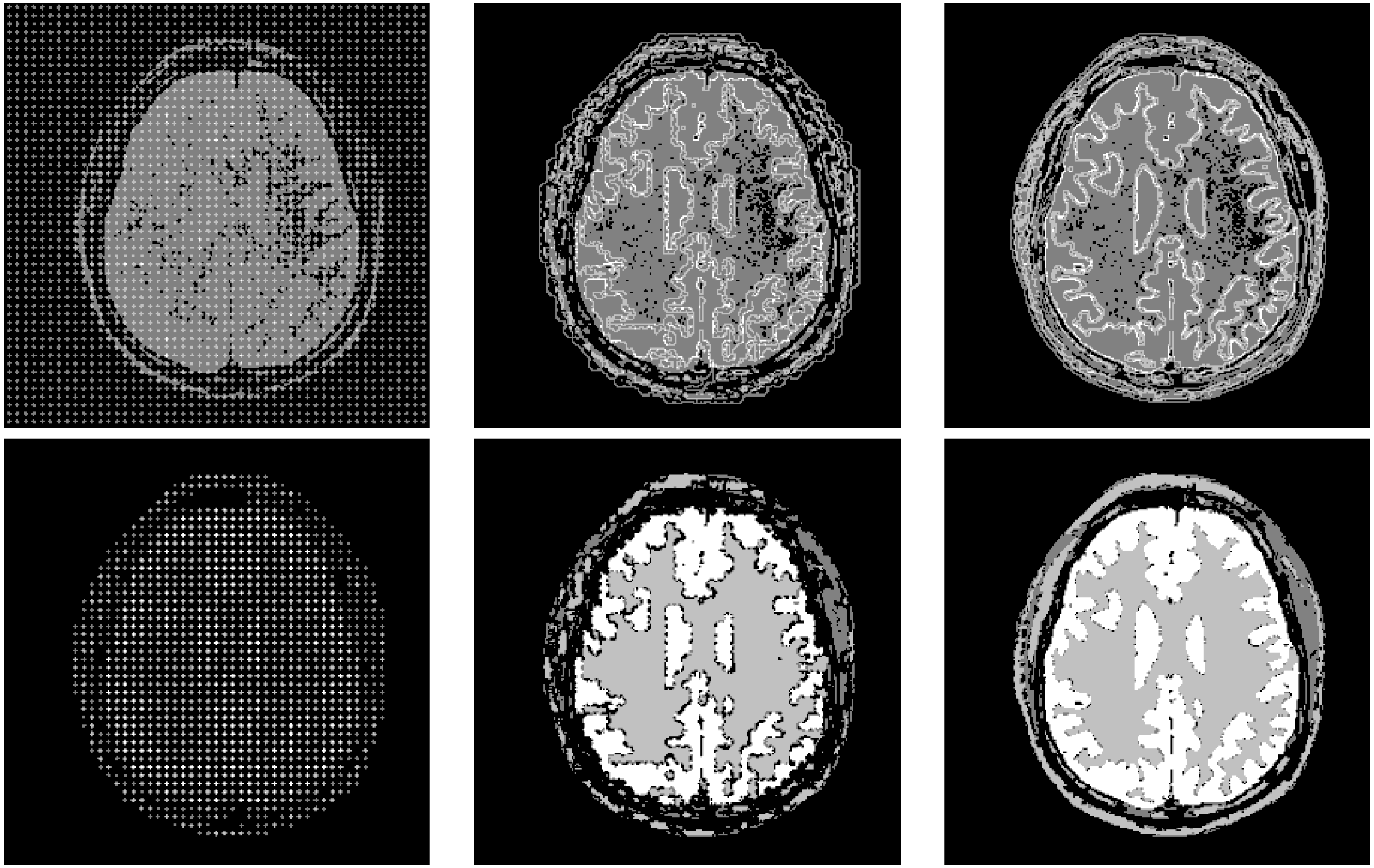
$$F_\alpha(\Phi_1,...,\Phi_K) = \underbrace{\sum_{i=1}^{K} e_i \int_\Omega H_\alpha(\Phi_i)\frac{(u_o-\mu_i)^2}{\sigma_i^2}dx}_{\text{data term (CONDITION B)}} + \underbrace{\sum_{i=1}^{K}\gamma_i \int_\Omega \delta_\alpha(\Phi_i)|\nabla\Phi_i|dx}_{\text{smooth boundaries (CONDITION C)}} + \underbrace{\frac{\lambda}{2}\int_\Omega\Big(\sum_{i=1}^{K}H_\alpha(\Phi_i)-1\Big)^2dx}_{\text{partition condition (CONDITION A)}}$$

- That when optimized leads to a set of equations that deforming simultaneously the initial curves according to:

$$\Phi_i^{t+1} = \Phi_i^t - dt\Big(\delta_\alpha(\Phi_i)\Big[e_i\frac{(u_0-\mu_i)^2}{\sigma_i^2} - \gamma_i div\Big(\frac{\nabla\Phi_i}{|\nabla\Phi_i|}\Big) + \lambda\Big(\sum_{i=1}^{K}H_\alpha(\Phi_i)-1\Big)\Big]\Big),$$

Multiphase Motion & Mumford-Shah

[samson-aubert-blanc-feraud:99]



initialization — iteration 10 — iteration 200

Multi-Phase Motion

- PROS
  - Taking the level set method to another level
  - Dealing with multiple (multi-component) objects, and multiple tasks
  - Introducing interactions between shape structures that evolve in parallel

- CONS
  - Computationally expensive
  - Difficult to guarantee convergence
  - Numerically unstable & hard to implement
  - Prior knowledge required on the number of classes and in some cases on their properties…

- PARTIAL SOLUTION: The multi-phase Chan-Vese model

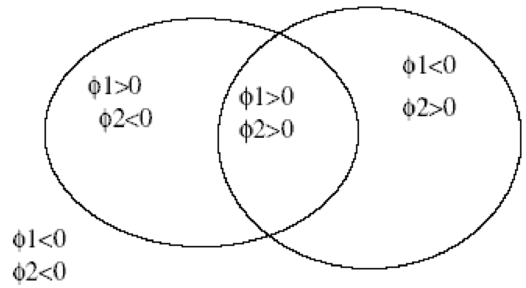
Multi-Phase Motion

[vese-chan:02]

- Introduce classification according to a combination of all level sets at a given pixel

LEVEL SET DICTIONARY

- Class 1: $H_\alpha(\phi_1)H_\alpha(\phi_2) = 1$
- Class 2: $H_\alpha(\phi_1)(1 - H_\alpha(\phi_2)) = 1$
- Class 3: $(1 - H_\alpha(\phi_1))H_\alpha(\phi_2) = 1$
- Class 4: $(1 - H_\alpha(\phi_1))(1 - H_\alpha(\phi_2)) = 1$



- And therefore by taking these products one can define a modified version of the mumford-shah approach to account with four classes while using two level set functions…

$$E(\phi_1,\phi_2,\mu) = \int\int H_\alpha(\phi_1)H_\alpha(\phi_2)(I-\mu_{11})^2 + \int\int H_\alpha(\phi_1)(1-H_\alpha(\phi_2))(I-\mu_{12})^2$$

$$\int\int (1-H_\alpha(\phi_1))H_\alpha(\phi_2)(I-\mu_{21})^2 + \int\int (1-H_\alpha(\phi_1))(1-H_\alpha(\phi_2))(I-\mu_{22})^2 + w\int\int \delta_\alpha(\phi_1)$$

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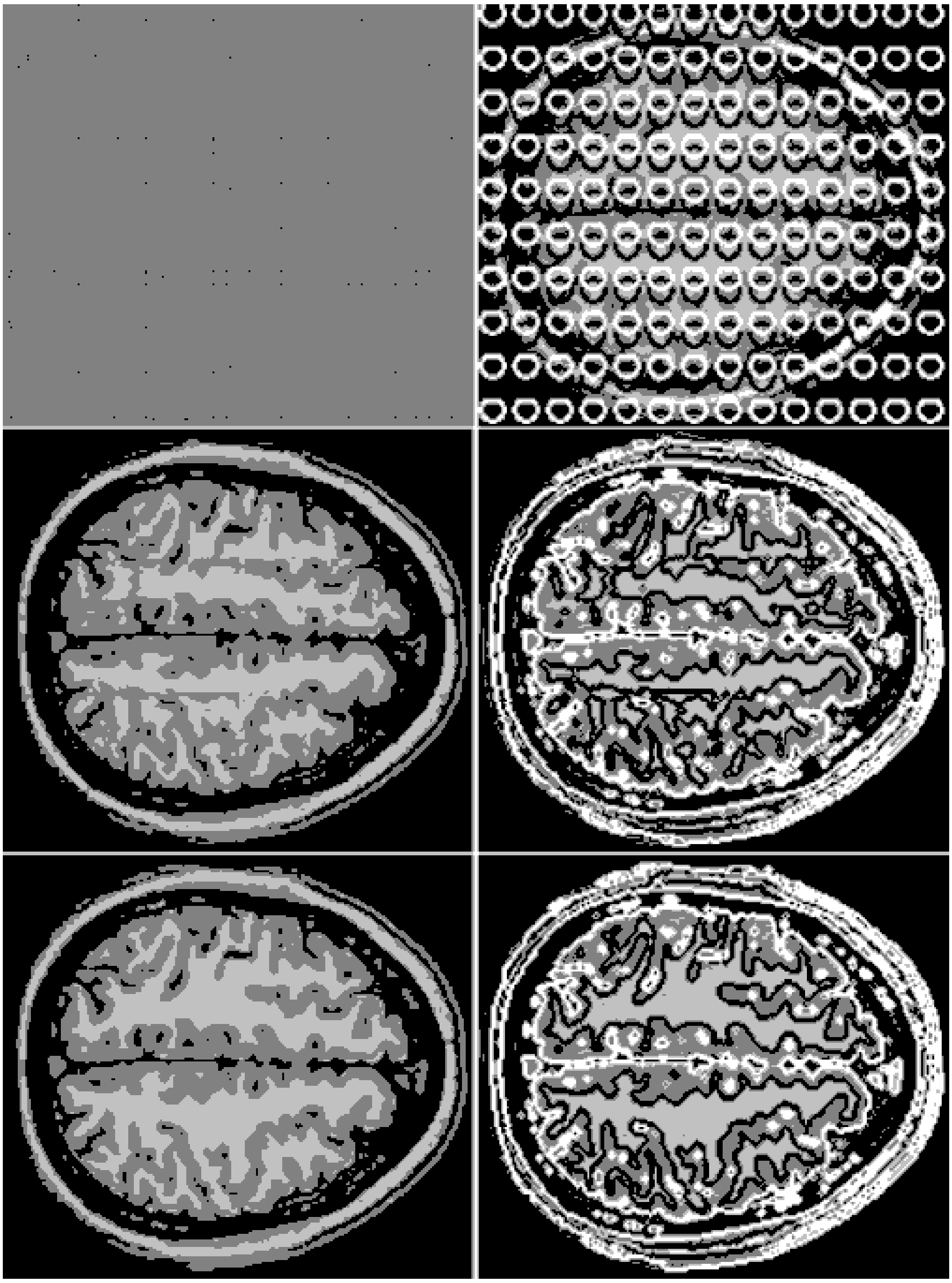

Multi-Phase Motion

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Multi-Phase Motion with more advanced data-driven terms

- The assumption of piece-wise constant is rather weak in particular in medical imaging…
- Several authors have proposed more advanced statistical formulations that are recovered "on the fly" to determine the statistics of each class
- The case of non-parametric approximations of the histogram within each region is a promising direction

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ATLANTIS

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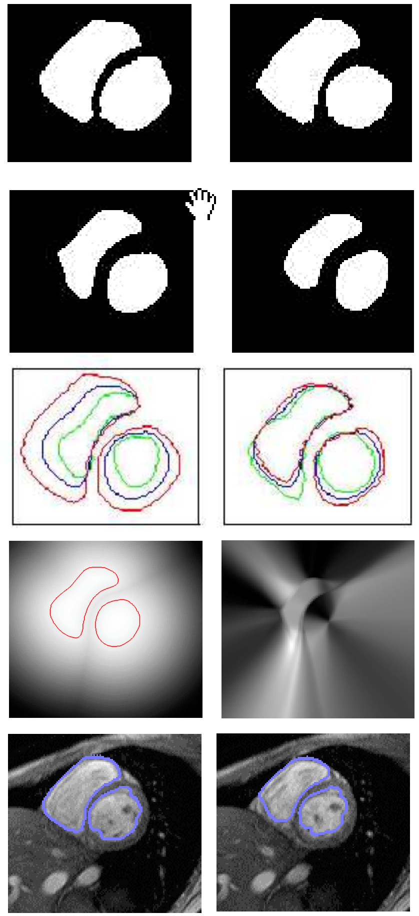
Knowledge-based Object Extraction

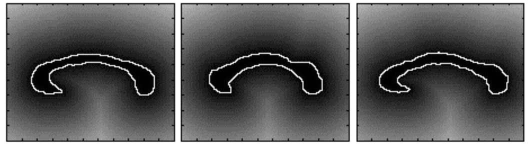
Knowledge-based Object Extraction

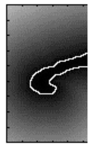
- Objective:
  recover from the image a structure
  of a particular – known to some extend
  – geometric form
- Methodology
  - Consider a set of training examples
  - Register these examples to a common pose
  - Construct a compact model that expresses the variability of the training set
  - Given a new image, recover the area where the underlying object looks like that one learnt
- Advantages of doing that on the LS space:
  - Preserve the implicit geometry
  - Account with multi-component objects…
  - … all wonderful staff you can do with the LS

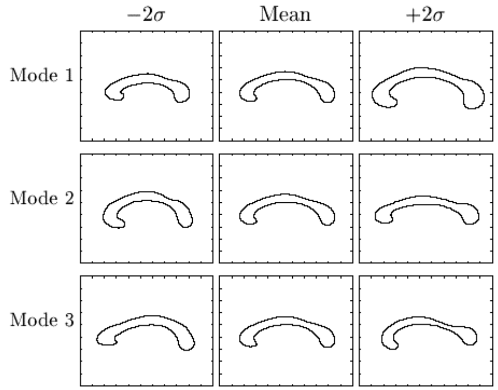


Knowledge-based Segmentation

[leventon-faugeras-grimson-etal:00]



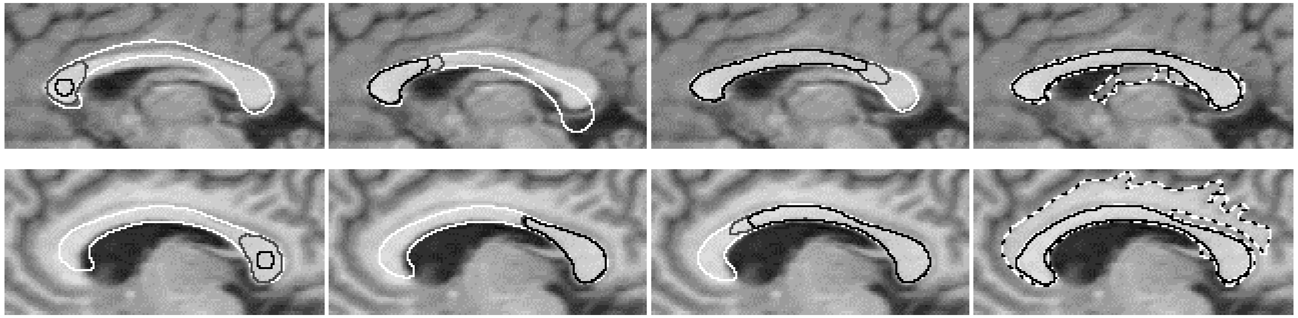




- Concept: Alternate between segmentation & imposing prior knowledge
  - Learn a Gaussian distribution of the shape to be recovered from a training set directly at the space of implicit functions
    - The elements of the training set are registered
    - A principal component analysis is use to recover the covariance matrix of probability density function of this set
  - ALTERNATE
    - Evolve a let set function according to the geodesic active contour
    - Given its current form, deform it locally using a MAP criterion so it fits better with the prior distribution
    - Until convergence...

Knowledge-based Segmentation

[leventon-faugeras-grimson-etal:00]



- Limitations:
  - Data driven & prior term are decoupled
  - Building density functions on high dimensional spaces is an ill posed problem,
  - Dealing with scale and pose variations (they are not explicitely addressed)

Knowledge-based Segmentation

[chen-etal:01]

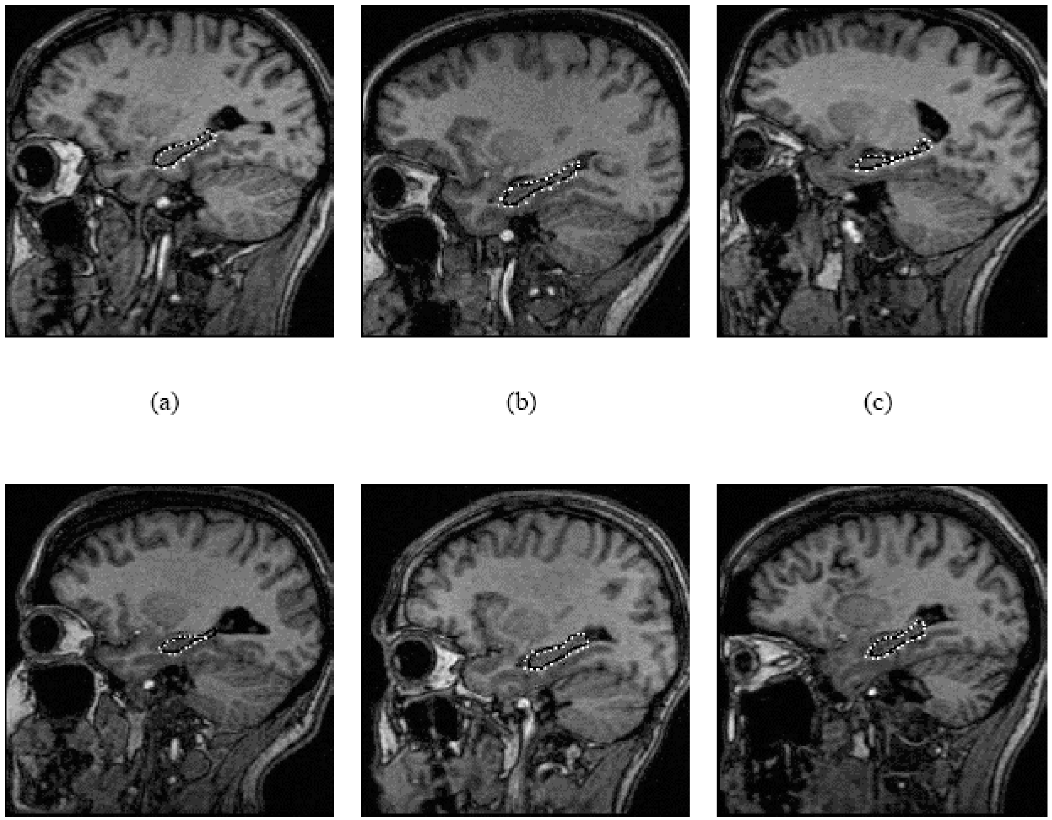
- Concept level:
  - Use an average model as prior in its implicit function
  - For a given curve find the transformation that projects it closer to the zero-level set of the implicit representation of the prior
  - For a given transformation evolve the curve locally towards better fitting with the prior…
  - Couple prior with the image driven term in a direct form…

$$\min_{u,\mu,R,T} \int_{\Omega} \delta(u)\{g(|\nabla I|) + \frac{\lambda}{2} d^2(\mu R x + T)\}|\nabla u|$$

- Issues to be addressed:
  - Model is very simplistic (average shape) – opposite to the leventon's case where it was too much complicated…
  - Estimation of the projection between the curve and the model space is tricky…not enough support…data term can be improved…

Knowledge-based Segmentation

[chen-etal:01]



(a) (b) (c)

(d) (e) (f)

Knowledge-based Segmentation

[tsai-yezzi-etal:01]

- At a concept level, prior knowledge is modeled through a Gaussian distribution on the space of distance functions by performing a singular value decomposition on the set of registered training set,
- The mumford-shah framework determined at space of the model is used to segment objects according to various data-driven terms
- The parameters of the projection are recovered at the same time with the segentation result…
  - *A more convenient approach than the one of Leventon-etal*
  - Which suffers from not comparing directly the structure that is recovered with the model…

Knowledge-based Segmentation

[paragios-rousson:02]

- Prior is imposed by direct comparison between the model and evolving contour modulo a similarity transformation…

- The model consists of a stochastic level set with two components,
  - A distance map that refers to the average model
  - And a confidence map that dictates the accuracy of the model

$$p_s(\phi) = \frac{1}{\sqrt{2\pi}\sigma_m(s)} e^{-\frac{(\phi - \phi_m(s))^2}{2\sigma_m(s)^2}}$$

- Objective: Recover a level set that pixel-wise looks like the prior modulo some transformation

Model Construction

$$p_s(\phi) = \frac{1}{\sqrt{2\pi}\sigma_m(s)} e^{-\frac{(\phi - \phi_m(s))^2}{2\sigma_m(s)^2}}$$

- From a training set recover the most representative model;
- If we assume N samples on the training set, then the distribution that expresses at a given point most of these samples is the one recovered through MAP

$$E(\phi_m(s), \sigma_m(s)) = -\log \sum_{i=1}^{N} p_s(\phi_i(s)) = \sum_{i=1}^{N} \left[\log(\sigma_m(s)) + \frac{(\phi_i - \phi_m(s))^2}{2\sigma_m(s)^2} \right]$$

- Where at a given pixel, we recover the mean and the variance that best describes the training set composed of implicit functions at this point, where the mean corresponds to the average value
- Constraints on the variance to be locally smooth is a natural assumption

$$E(\phi_m, \sigma_m) = \alpha \sum_{i=1}^{N} \int\int_{\Omega} \left[\log(\sigma_m) + \frac{(\phi_i - \phi_m)^2}{2\sigma_m^2} \right] d\Omega + \int\int_{\Omega} \psi\left(\nabla \sigma_m\right) \, d\Omega$$

Model Construction (continued)

- The calculus of variations can lead to the estimation of the mean and variance (confidence measure) of the model at et each pixel,
- However, the resulting model will not be an implicit function in the sense of distance transform (averaging distance transforms doesn't necessary produce one)
- One can seek for a solution of the previously defined objective function subject to the constraint the "means" field forms a distance transform using Lagrange multipliers...
- An alternative is to consider the process in repeated steps where first a solution that fits the data is recovered and then is projected to the space of distance functions...

$$\frac{d}{d\tau}\phi_m = \mathrm{sgn}\left(\phi_m^0\right)\left(1 - |\nabla\phi_m|\right)$$

Imposing the (Static) Prior

- Define/recover a morphing function "A" that creates correspondence between the model and the prior

$$\phi(;\tau) = \phi_m(\mathcal{A}(;\tau))$$

- In the absence of scale variations, and in the case of global morphing functions one can compare the evolving contour with the model according to

$$E(\phi, \mathcal{A}) = \int\int_\Omega \delta_\epsilon(\phi)(\phi - \phi_m(\mathcal{A}))^2 d\Omega$$

- That modulo the morphing function will evolve the contours towards a better fit with the model

- One can prove that scale variations introduce a multiplicative factor and they have to be explicitly taken into account

Static Prior (continued)

$$E(\phi, \mathcal{A}) = \int\int_{\Omega} \delta_{\epsilon}(\phi)(\mathcal{S}\phi - \phi_m(\mathcal{A}))^2 d\Omega$$

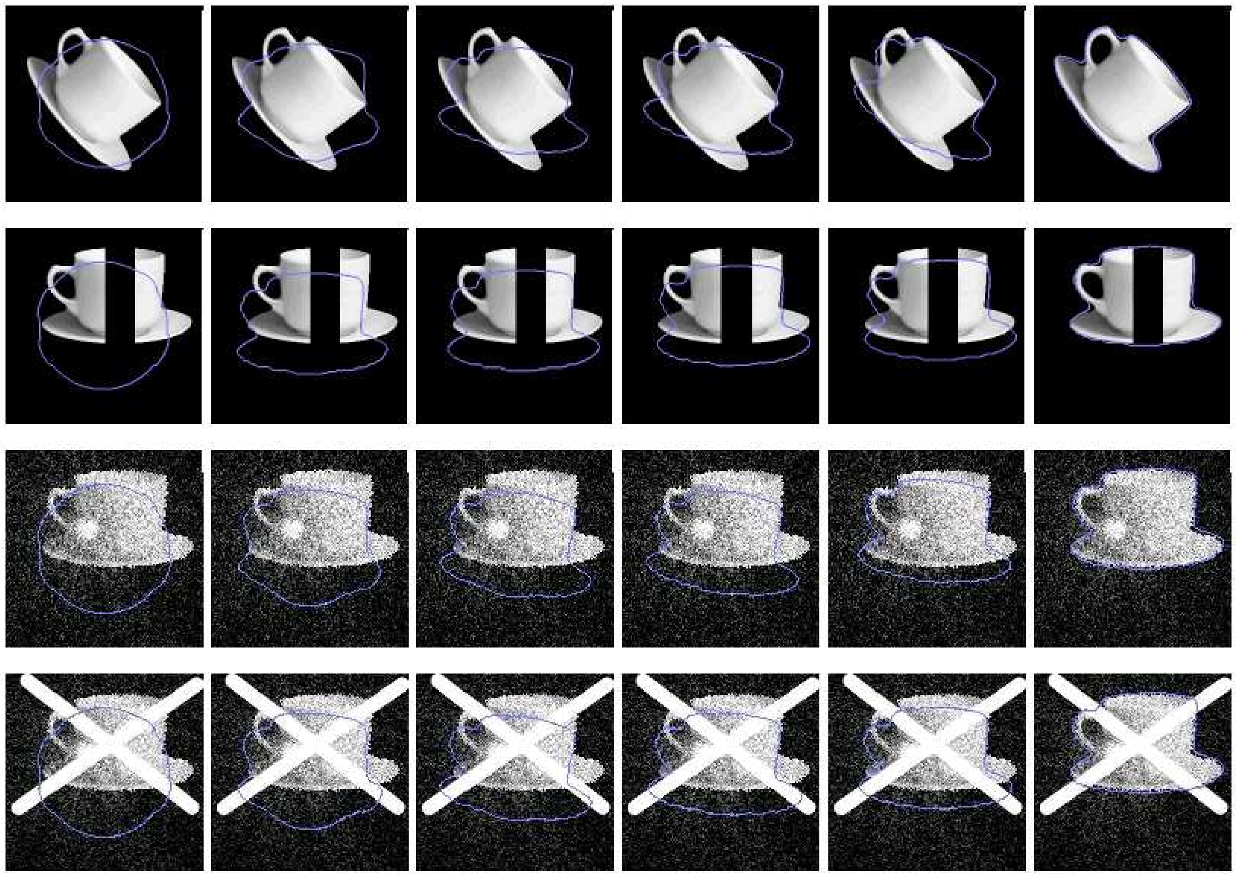
- Where the unknowns are the morphing function and the position of the level set

- Calculus of variations with respect to the position of the interface are straightforward:

$$\frac{d}{d\tau}\phi = -2\underbrace{\delta_{\epsilon}(\phi)\mathcal{S}(\mathcal{S}\phi - \phi_m(\mathcal{A}))}_{shape\ consistency\ force} - \underbrace{\left[\frac{\partial}{\partial\phi}\delta_{\epsilon}(\phi)\right](\mathcal{S}\phi - \phi_m(\mathcal{A}))^2}_{area\ force}$$

- The second term is a constant inflation term aims at minimizing the area of the contour and eventually the cost function and can be ignored…since it has no physical meaning.

Static Prior, Concept Demonstration



Static Prior (continued)

- One can also optimize the cost function with respect to the unknown parameters of the morphing function

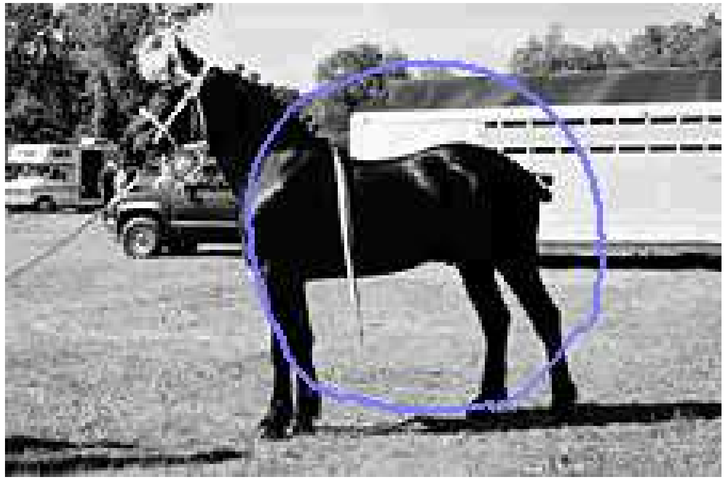
$$\frac{d}{d\tau}\Theta = -2\int\int_{\Omega} \delta_{\epsilon}(\phi)(\mathcal{S}\phi - \phi_m(\mathcal{A}))(-\nabla\phi_m(\mathcal{A}) \cdot \frac{\partial}{\partial\Theta}\mathcal{A})\; d\Omega$$

$$\frac{d}{d\tau}\mathcal{S} = -2\int\int_{\Omega} \delta_{\epsilon}(\phi)(\mathcal{S}\phi - \phi_m(\mathcal{A}))(-\phi - \nabla\phi_m(\mathcal{A}) \cdot \frac{\partial}{\partial\mathcal{S}}\mathcal{A})\; d\Omega$$

$$\frac{d}{d\tau}\begin{bmatrix}\mathcal{T}x \\ \mathcal{T}y\end{bmatrix} = -2\int\int_{\Omega} \delta_{\epsilon}(\phi)(\mathcal{S}\phi - \phi_m(\mathcal{A}))(-\nabla\phi_m(\mathcal{A}) \cdot \frac{\partial}{\partial\begin{bmatrix}\mathcal{T}x \\ \mathcal{T}y\end{bmatrix}}\mathcal{A})\; d\Omega$$

- Leading to a nice "self-sufficient" system of motion equations that update the global registration parameters between the evolving curve and the model
- However, the variability of the model was not considered up to this point and areas with high uncertainties will have the same impact on the process

Some Results (non-medical)



Taking Into Account the Model Uncertainties

- Maximizing the joint posterior (segmentation/morphing) is a quite attractive criterion in "inferencing"

$$p(\mathcal{A},\phi|\phi_m) = \frac{p(\phi_m|\mathcal{A},\phi)}{p(\phi_m)}p(\phi,\mathcal{A}) = \frac{p(\phi_m(\mathcal{A})|\phi)}{p(\phi_m)}p(\phi,\mathcal{A})$$

- Where the Bayes rule was considered and given that the probability for a given prior model is fixed and we can assume that all (segmentation/morphing) solutions are equally probable, we get

$$p(\phi_m(\mathcal{A})|\phi) = \prod_{\omega\in\Omega} p(\phi_m(\mathcal{A}(\omega))|\mathcal{S}\phi(\omega))$$

- Under the assumption of independence...within pixels...and then finding the optimal implicit function and its morphing transformations is equivalent with

$$E(\phi,\mathcal{A}) = -\log\left[\prod_{\omega\in\Omega} p(\phi_m(\mathcal{A}(\omega))|\mathcal{S}\phi(\omega))\right] = -\int\int_{\Omega}\log(p_{\omega}(\mathcal{S}\phi(\omega)))d\Omega$$

Taking Into Account the Model Uncertainties

- That can be further developed using the Gaussian nature of the model distribution at each image pixel

$$E(\phi, \mathcal{A}) = \int\int_{\Omega} \left(\log(\sigma_m(\mathcal{A})) + \frac{(\mathcal{S}\phi - \phi_m(\mathcal{A}))^2}{2\sigma_m(\mathcal{A})^2} \right) d\Omega$$

- A term that aims at recovering a transformation and a level set that when projected to the model, it is projected to areas with low variance (high confidence)
- A term that aims at minimizing the actual distance between the level set function and the model and is scaled according to the model confidence…
    - would prefer have a better match between the model and level set in areas where the variability is low,
    - while in areas with important deviation of the training set, this term will be less important

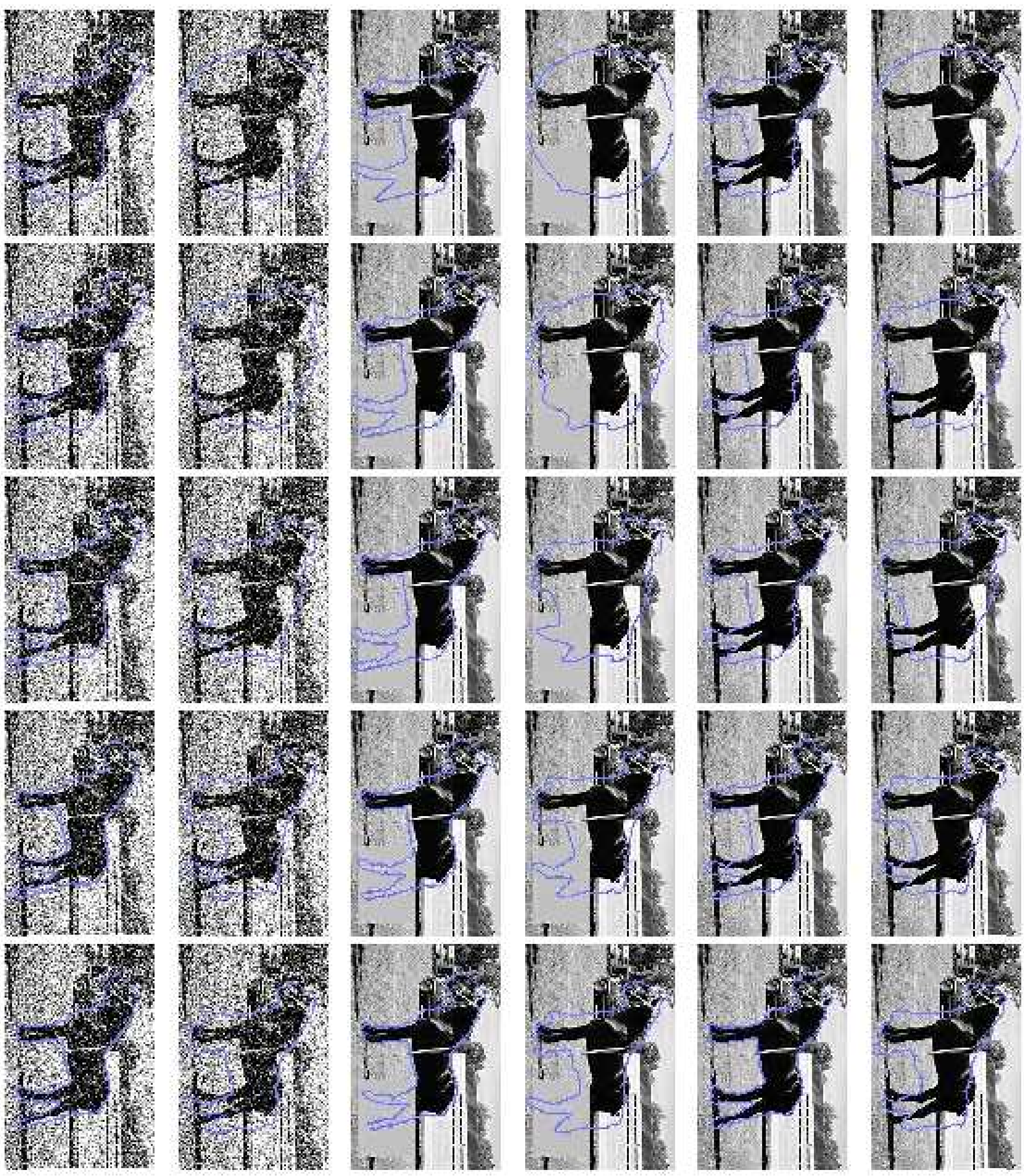
Taking the derivatives…

- Calculus of variations regarding the level set and the morphing function:

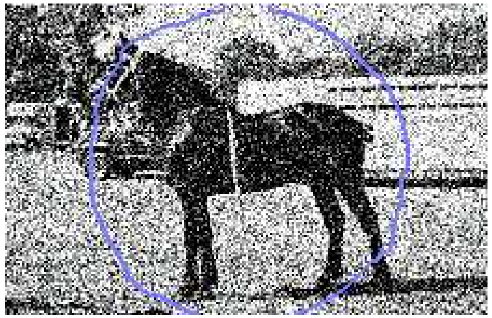
$$\frac{d}{d\tau}\phi = -\underbrace{\left[\frac{\partial}{\partial\phi}\delta_\epsilon(\phi)\right]\left(\log(\sigma_m(\mathcal{A})) + \frac{(\mathcal{S}\phi - \phi_m(\mathcal{A}))^2}{\sigma_m(\mathcal{A})^2}\right)}_{area\ force} \underbrace{-2\,\delta_\epsilon(\phi)\mathcal{S}\frac{(\mathcal{S}\phi - \phi_m(\mathcal{A}))}{\sigma_m(\mathcal{A})^2}}_{shape\ consistency\ force}$$

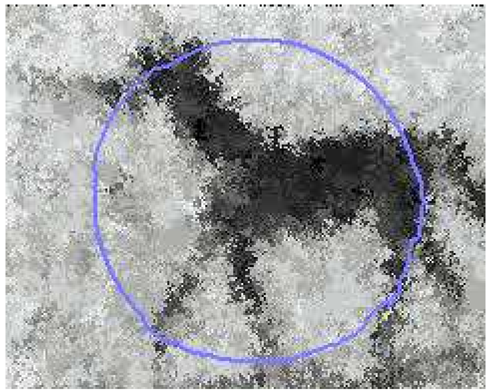
- The level set deformation flow consists of two terms:
  - that is a constant deflation force (when the level set function collapses, eventually the cost function reaches the lowest potential)
  - An adaptive balloon (directional/magnitude-wise) force that inflates/deflates the level set so it fits better with the prior after its projection to the model space…In areas with high variance this term become less significant and data-terms guide the level set to the real object boundaries…

Comparative Results...

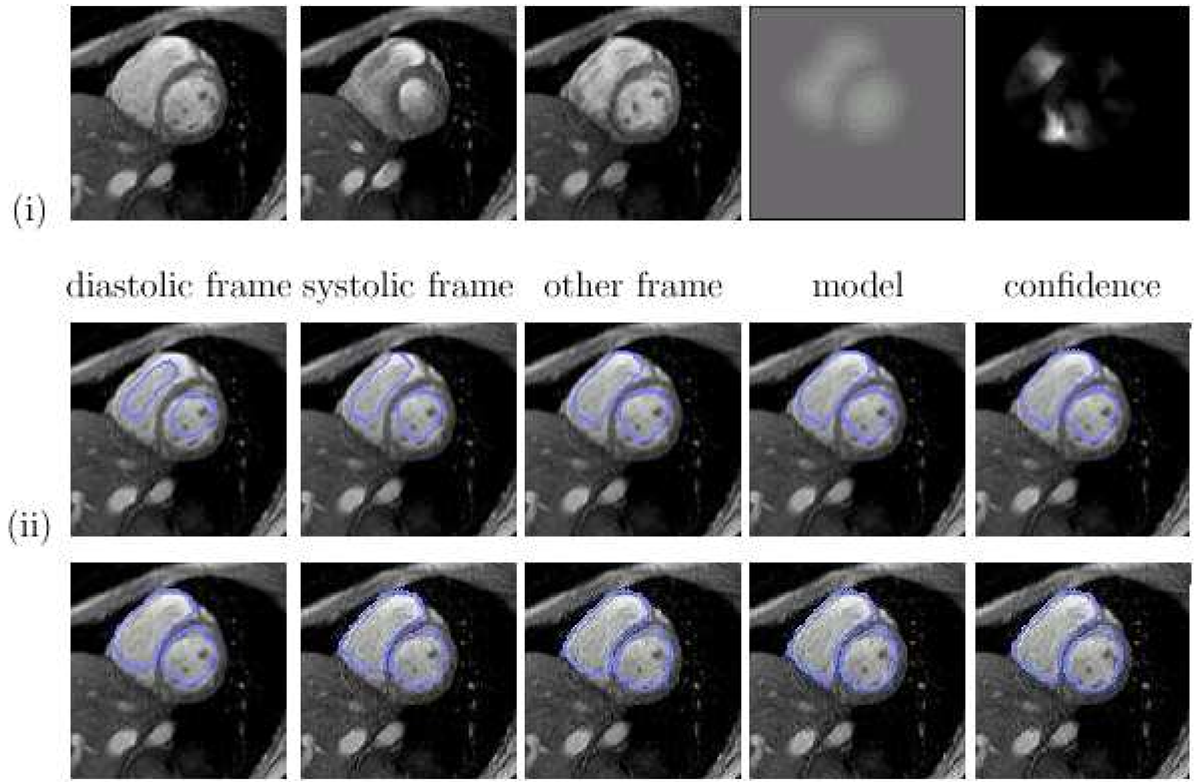


Some Videos...(again non-medical)





Some medical results



Implicit Active Shapes

[rousson-paragios:03]

- The Active Shape Model of Cootes et al. is quite popular to object extraction. Such modeling consists of the following steps:

- Let us consider a training set $\mathcal{S}_i$ of N **registered surfaces** (implicit representations can also be used for registration [4]). Distance maps are computed for each surface:

$$\mathcal{S}_i \rightarrow \phi_i, \quad i = 1..N$$

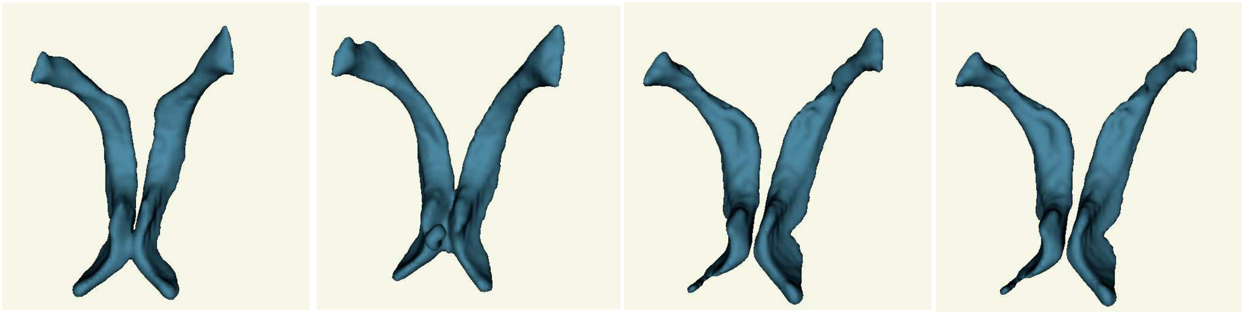
- The samples N are centered with respect to the average representation :

$$\psi_i = \phi_i - \phi_{\mathcal{M}}$$

Implicit Active Shapes

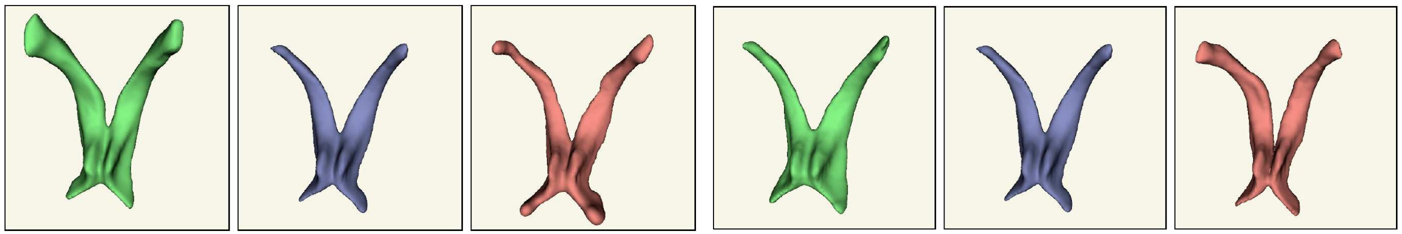
[rousson-paragios:03]

- Training set:

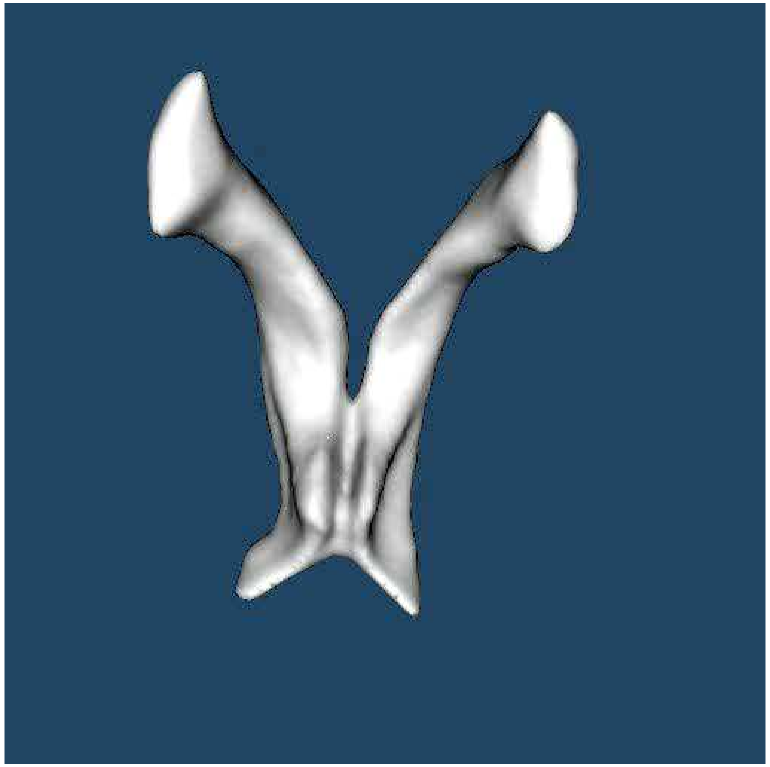


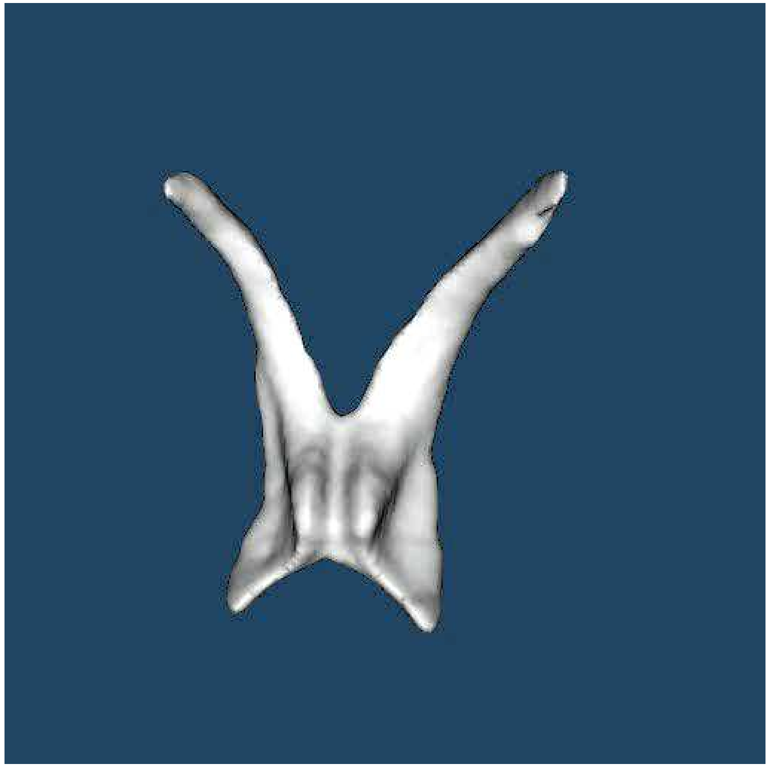
- The **principal modes of variation** U_j are recovered through Principal Component Analysis (PCA). A new shape ϕ can be generated from the (m) retained modes:

$$\phi = \phi_{\mathcal{M}} + \sum_{j=1}^{m} \lambda_j U_j$$



The model...





The prior

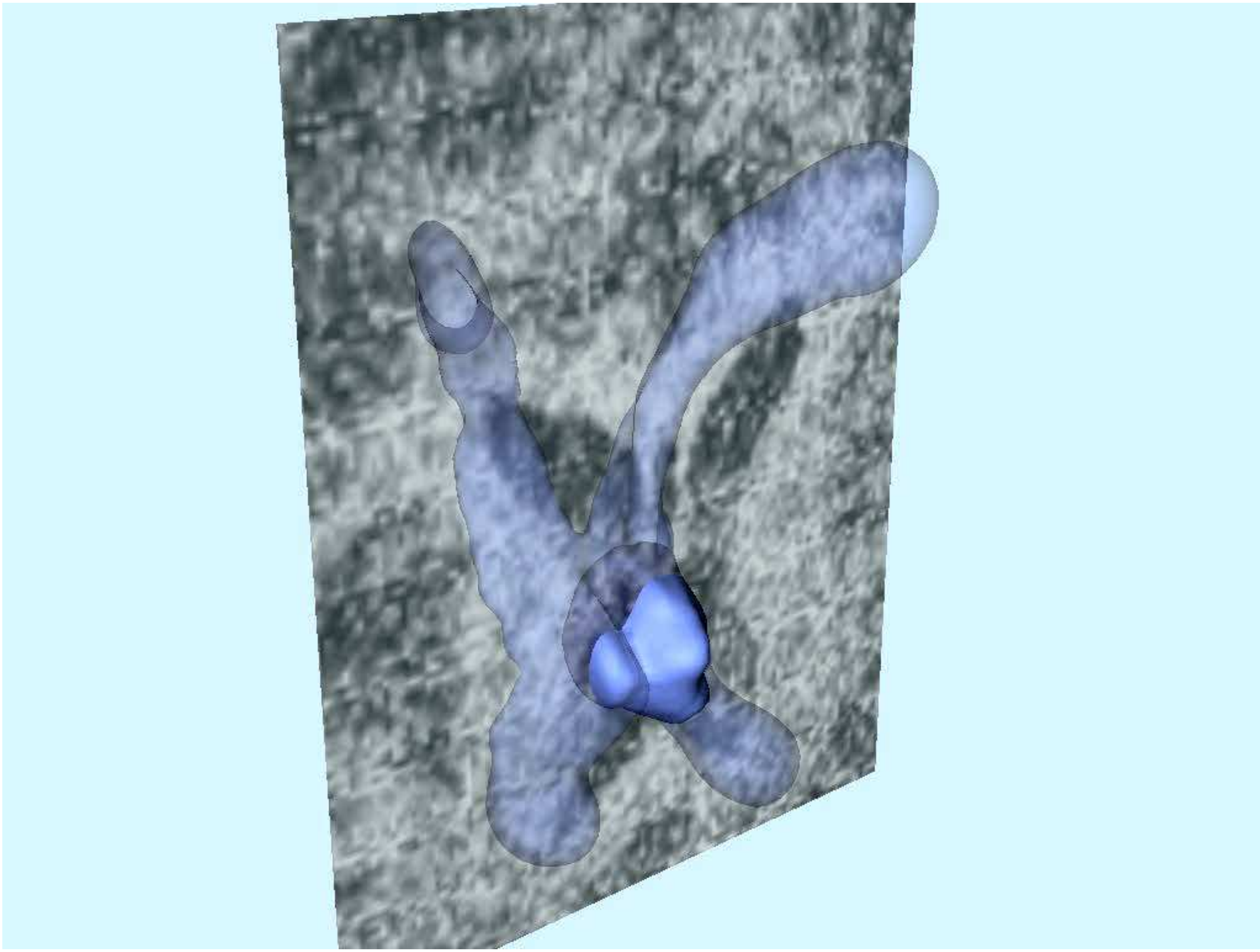
- A level set function that has minimal distance from a linear from the model space…

$$E(\phi, \mathcal{A}, \alpha) = \iint_{\Omega} \delta_{\epsilon}(\phi) \left(\mathcal{S}\phi - \left(\phi_{\mathcal{M}}(\mathcal{A}) + \sum_{j=1}^{m} \lambda_j \, U_j(\mathcal{A}) \right) \right)^2 d\Omega$$

- The unknown consist of:
  - The form of the implicit function
  - The global transformation between the average mode and the image,
  - The set of linear coefficients that when applied to the set of basis functions provides the optimal match of the current contour with the model space

- And are recovered in a straightforward manner using a gradient descent method…

Some nice results...



Conclusions

- PROS
  - Elegant tool to track moving interfaces
  - Implicit Curve Parameterization & estimation of the geometric Properties
  - Able to account with topological changes, able to describe multi-component objects

- CONS
  - Computational complexity
  - Numerical approximations, redundancy
  - Open Curves, sorry we CANNOT do anything about that…

Http://cermics.enpc.fr/~paragios/book/book.html

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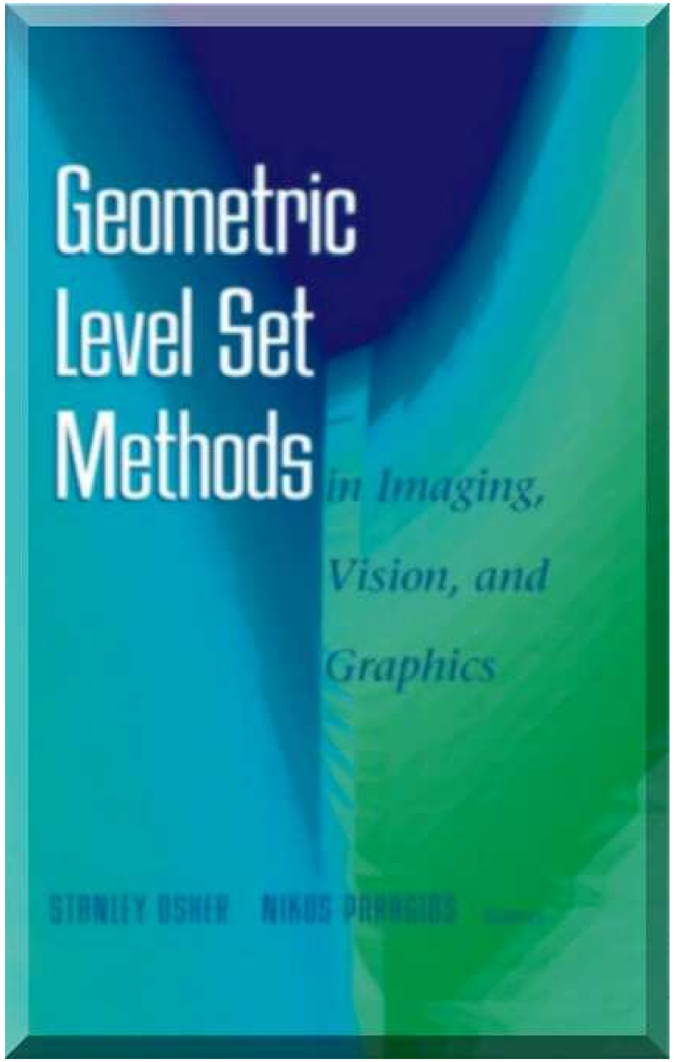

Resources

- Books
  - James Sethian (1996,1999): Level Set & Fast Marching Methods, Cambridge, Introductory.
  - Stan Osher & Ronald Fedkiw (2002): Level Set Methods and Dynamic Implicit Surfaces, Springer, Introductory.
  - Stan Osher & Nikos Paragios (2003): Geometric Level Set in Imaging Vision and Graphics, Springer, ...Mostly Vision...bit advanced...

- People [non-exclusive list]
  - Laurent Cohen (medical), David Breen (graphics), Eric Grimson (medical), Olivier Faugeras (stereo), Renaud Keriven (stereo, segmentation), Ron Kimmel (segmentation, shape from shading, tracking), Jerry Prince (topology preserving), Guillermo Sapiro (segmentation, tracking, implicit surfaces), James Sethian, Baba Vemuri (Diffusion, Segmentation, Registration) Joachim Weickert (diffusion, segmentation), Ross Whitaker (Graphics), Allan Willsky (medical), Anthony Yezzi (medical), ...